14.385 Nonlinear Econometric Analysis

Matching

Maximilian Kasy

Department of Economics, MIT

Fall 2022

Outline

- Matching estimators for
  - average treatment effect,
  - average treatment effect on the treated.
- Curse of dimensionality and matching bias.
- Alternative estimators:
  - Inverse probability weighting.
  - Regression.
  - Double robust (next part of class).
- Causal graphs.

Takeaways for this part of class

- Identification under conditional independence is based on comparing the outcomes of units that “look the same.”
- Matching implements this intuition.
- In high dimensions, it can be difficult finding good matches.
- There are numerous alternative estimators based on the same identification assumption (matching, weighting, regression, double robust).
- The backdoor criterion, based on causal graphs, provides a useful check whether conditional independence is plausible.

Identification under Selection on Observables

Assumption

1. $(Y_1, Y_0) \perp D|X$ (selection on observables)
2. $0 < \Pr(D = 1|X) < 1$ with probability one (common support)

- Given selection on observables we have

$$\begin{aligned} E[Y_1 - Y_0|X] &= E[Y_1 - Y_0|X, D = 1] \\ &= E[Y|X, D = 1] - E[Y|X, D = 0] \end{aligned}$$

- Therefore, under the common support condition:

$$\begin{aligned} \alpha_{ATE} &= E[Y_1 - Y_0] = \int E[Y_1 - Y_0|X]\, dP(X) \\ &= \int \left(E[Y|X, D = 1] - E[Y|X, D = 0]\right) dP(X) \end{aligned}$$

Identification under Selection on Observables

Assumption

1. $(Y_1, Y_0) \perp D|X$ (selection on observables)
2. $0 < \Pr(D = 1|X) < 1$ with probability one (common support)

- Similarly,

$$\begin{aligned} \alpha_{ATET} &= E[Y_1 - Y_0|D = 1] \\ &= \int \left(E[Y|X, D = 1] - E[Y|X, D = 0] \right) dP(X|D = 1) \end{aligned}$$

- To identify α_{ATET} the selection on observables and common support conditions can be relaxed to:
  - $Y_0 \perp D|X$
  - $\Pr(D = 1|X) < 1$ (with $\Pr(D = 1) > 0$)

Subclassification Estimator

- The identification result is:

$$\begin{aligned}\alpha_{ATE} &= \int \left(E[Y|X,D=1]-E[Y|X,D=0]\right) dP(X) \\ \alpha_{ATET} &= \int \left(E[Y|X,D=1]-E[Y|X,D=0]\right) dP(X|D=1)\end{aligned}$$

- Assume X takes on K different cells $\{X^1,...,X^k,...,X^K\}$.

Subclassification Estimator

- The identification result is:

$$\alpha_{ATE} = \int \left(E[Y|X,D=1] - E[Y|X,D=0]\right) dP(X)$$
$$\alpha_{ATET} = \int \left(E[Y|X,D=1] - E[Y|X,D=0]\right) dP(X|D=1)$$

- Assume X takes on K different cells $\{X^1,...,X^k,...,X^K\}$.
- The analogy principle suggests the following estimators:

$$\widehat{\alpha}_{ATE} = \sum_{k=1}^{K} (\bar{Y}_1^k - \bar{Y}_0^k) \cdot \left(\frac{N^k}{N}\right) \; ; \; \widehat{\alpha}_{ATET} = \sum_{k=1}^{K} (\bar{Y}_1^k - \bar{Y}_0^k) \cdot \left(\frac{N_1^k}{N_1}\right)$$

  - N^k is # of obs. and N_1^k is # of treated obs. in cell k
  - $\bar{Y}_1^k$ is mean outcome for the treated in cell k
  - $\bar{Y}_0^k$ is mean outcome for the untreated in cell k

Subclassification and the “Curse of Dimensionality”

- Subclassification becomes unfeasible with many covariates
- Assume we have k covariates and divide each of them into 3 coarse categories (e.g., age could be “young”, “middle age” or “old”, and income could be “low”, “medium” or “high”).
- The number of subclassification cells is 3^k. For $k = 10$, we obtain $3^{10} = 59049$
- Many cells may contain only treated or untreated observations, so we cannot use subclassification
- Subclassification is also problematic if the cells are “too coarse”. But using “finer” cells worsens the curse of dimensionality problem: e.g., using 10 variables and 5 categories for each variable we obtain $5^{10} = 9765625$

Matching

- We could also estimate α_{ATET} by constructing a comparison sample of untreated units with the same characteristics as the sample of treated units.

- This can be easily accomplished **matching** treated and untreated units with the same characteristics.

- If all matches are perfect (no matching discrepancies) the resulting estimator is identical to the subclassification estimator.

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
Average:		

Matching: An Ideal Example

Trainees		
unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees		
unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample		
unit	age	earnings
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
Average:		

Matching: An Ideal Example

Trainees		
unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees		
unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample		
unit	age	earnings
8	28	8800
14	34	24200
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
Average:		

Matching: An Ideal Example

Trainees		
unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees		
unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample		
unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
17	29	6200
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
17	29	6200
9,16	24	17700
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
17	29	6200
9,16	24	17700
15	25	23300
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
17	29	6200
9,16	24	17700
15	25	23300
1	43	20900
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
17	29	6200
9,16	24	17700
15	25	23300
1	43	20900
8	28	8800
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
17	29	6200
9,16	24	17700
15	25	23300
1	43	20900
8	28	8800
4	27	9300
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
17	29	6200
9,16	24	17700
15	25	23300
1	43	20900
8	28	8800
4	27	9300
8	28	8800
Average:		

Matching: An Ideal Example

Trainees

unit	age	earnings
1	28	17700
2	34	10200
3	29	14400
4	25	20800
5	29	6100
6	23	28600
7	33	21900
8	27	28800
9	31	20300
10	26	28100
11	25	9400
12	27	14300
13	29	12500
14	24	19700
15	25	10100
16	43	10700
17	28	11500
18	27	10700
19	28	16300
Average:	28.5	16426

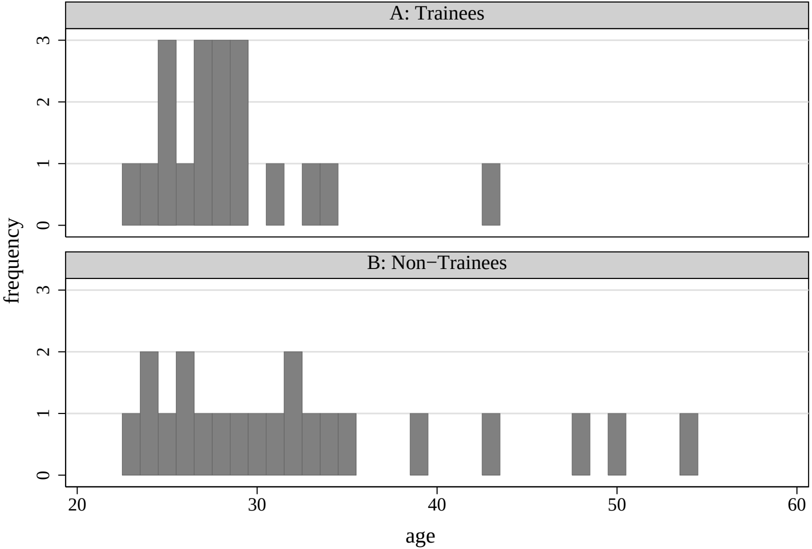
Non-Trainees

unit	age	earnings
1	43	20900
2	50	31000
3	30	21000
4	27	9300
5	54	41100
6	48	29800
7	39	42000
8	28	8800
9	24	25500
10	33	15500
11	26	400
12	31	26600
13	26	16500
14	34	24200
15	25	23300
16	24	9700
17	29	6200
18	35	30200
19	32	17800
20	23	9500
21	32	25900
Average:	33	20724

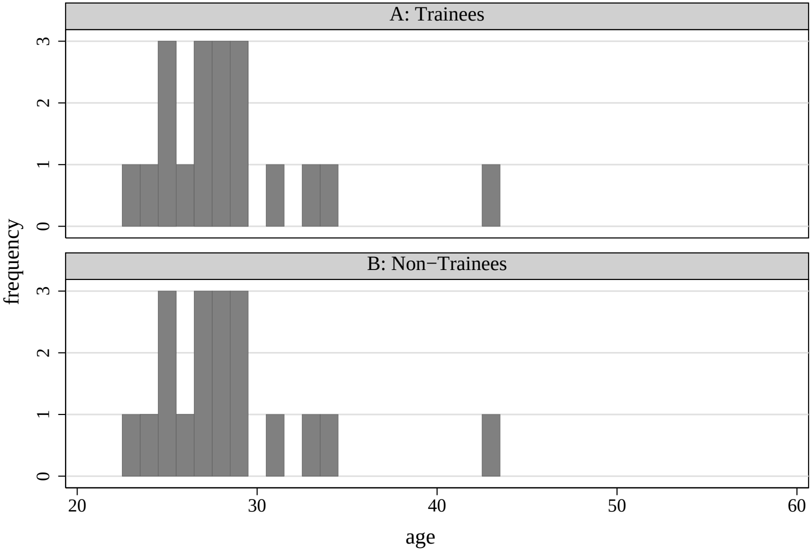
Matched Sample

unit	age	earnings
8	28	8800
14	34	24200
17	29	6200
15	25	23300
17	29	6200
20	23	9500
10	33	15500
4	27	9300
12	31	26600
11,13	26	8450
15	25	23300
4	27	9300
17	29	6200
9,16	24	17700
15	25	23300
1	43	20900
8	28	8800
4	27	9300
8	28	8800
Average:	28.5	13982

Age Distribution: Before Matching



Age Distribution: After Matching



Treatment Effect Estimates

Difference in average earnings between trainees and non-trainees:

- Before matching

$$16426 - 20724 = -4298$$

- After matching:

$$16426 - 13982 = 2444$$

Matching

- Perfect matches are often not available.
- In that case, a matching estimator of α_{ATET} can be constructed as:

$$\widehat{\alpha}_{ATET} = \frac{1}{N_1} \sum_{D_i=1} \left(Y_i - Y_{j(i)} \right)$$

where $Y_{j(i)}$ is the outcome of an untreated observation such that $X_{j(i)}$ is the **closest** value to X_i among the untreated observations.

Matching

- Perfect matches are often not available.
- In that case, a matching estimator of α_{ATET} can be constructed as:

$$\widehat{\alpha}_{ATET} = \frac{1}{N_1} \sum_{D_i=1} \left(Y_i - Y_{j(i)} \right)$$

where $Y_{j(i)}$ is the outcome of an untreated observation such that $X_{j(i)}$ is the **closest** value to X_i among the untreated observations.

- We can also use the average for M closest matches:

$$\widehat{\alpha}_{ATET} = \frac{1}{N_1} \sum_{D_i=1} \left\{ Y_i - \left(\frac{1}{M} \sum_{m=1}^{M} Y_{j_m(i)} \right) \right\}$$

- Works well when we can find good matches for each treated unit, so M is usually small (typically, $M = 1$ or $M = 2$).

Matching

- We can also use matching to estimate α_{ATE}. In that case, we match in both directions:
  1. If observation i is treated, we impute Y_{0i} using untreated matchs, $\{Y_{j_1(i)}, \dots, Y_{j_M(i)}\}$
  2. If observation i is untreated, we impute Y_{1i} using treated matchs, $\{Y_{j_1(i)}, \dots, Y_{j_M(i)}\}$
- The estimator is:

$$\widehat{\alpha}_{ATE} = \frac{1}{N}\sum_{i=1}^{N}(2D_i - 1)\left\{Y_i - \left(\frac{1}{M}\sum_{m=1}^{M} Y_{j_m(i)}\right)\right\}$$

Matching: Distance Metric

- When the vector of matching covariates,

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_k \end{pmatrix},$$

  has more than one dimension ($k > 1$) we need to define a **distance metric** to measure "closeness".

- The usual **Euclidean distance** is:

$$\|X_i - X_j\| = \sqrt{(X_i - X_j)'(X_i - X_j)} = \sqrt{\sum_{n=1}^{k} (X_{ni} - X_{nj})^2}.$$

- The Euclidean distance is not invariant to changes in the scale of the X's.

- For this reason, we often use alternative distances that are invariant to changes in scale.

Matching: Distance Metric

- A commonly used distance is the **normalized Euclidean distance**:

$$\|X_i - X_j\| = \sqrt{(X_i - X_j)' \widehat{V}^{-1} (X_i - X_j)}$$

where

$$\widehat{V} = \begin{pmatrix} \widehat{\sigma}_1^2 & 0 & \cdots & 0 \\ 0 & \widehat{\sigma}_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \widehat{\sigma}_k^2 \end{pmatrix}.$$

- The normalized Euclidean distance is equal to:

$$\|X_i - X_j\| = \sqrt{\sum_{n=1}^{k} \frac{(X_{ni} - X_{nj})^2}{\widehat{\sigma}_n^2}}.$$

- Changes in the scale of X_{ni} affect also $\widehat{\sigma}_n$, and the normalized Euclidean distance does not change.

Matching: Distance Metric

- Another popular scale-invariant distance is the **Mahalanobis distance**:

$$\|X_i - X_j\| = \sqrt{(X_i - X_j)'\widehat{\Sigma}_X^{-1}(X_i - X_j)},$$

where $\widehat{\Sigma}_X$ is the sample variance-covariance matrix of X.

- We can also define arbitrary distances:

$$\|X_i - X_j\| = \sqrt{\sum_{n=1}^{k} \omega_n \cdot (X_{ni} - X_{nj})^2}$$

(with all $\omega_n \geq 0$) so that we assign large ω_n's to those covariates that we want to match particularly well.

Matching and the Curse of Dimensionality

- Matching discrepancies $\|X_i - X_{j(i)}\|$ tend to increase with k, the dimension of X
- Matching discrepancies converge to zero. But they converge very slowly if k is large
- Mathematically, it can be shown that $\|X_i - X_{j(i)}\|$ converges to zero at the same rate as $\dfrac{1}{N^{1/k}}$
- It is difficult to find good matches in large dimensions: you need many observations if k is large

Matching: Bias Problem

- Let

$$\widehat{\alpha}_{ATET} = \frac{1}{N_1}\sum_{D_i=1}(Y_i - Y_{j(i)}),$$

where $X_i \simeq X_{j(i)}$ and $D_{j(i)} = 0$.

- Let

$$\begin{aligned}
\mu_0(x) &= E[Y|X=x, D=0] = E[Y_0|X=x],\\
\mu_1(x) &= E[Y|X=x, D=1] = E[Y_1|X=x],\\
Y_i &= \mu_{D_i}(X_i) + \varepsilon_i.
\end{aligned}$$

- Then,

$$\begin{aligned}
\widehat{\alpha}_{ATET} &= \frac{1}{N_1}\sum_{D_i=1}\Big((\mu_1(X_i)+\varepsilon_i) - (\mu_0(X_{j(i)}) + \varepsilon_{j(i)})\Big)\\
&= \frac{1}{N_1}\sum_{D_i=1}(\mu_1(X_i) - \mu_0(X_{j(i)})) + \frac{1}{N_1}\sum_{D_i=1}(\varepsilon_i - \varepsilon_{j(i)}).
\end{aligned}$$

Matching: Bias Problem

$$
\begin{aligned}
\widehat{\alpha}_{ATET} - \alpha_{ATET} &= \frac{1}{N_1} \sum_{D_i=1} \left(\mu_1(X_i) - \mu_0(X_{j(i)}) - \alpha_{ATET} \right) \\
&+ \frac{1}{N_1} \sum_{D_i=1} \left(\varepsilon_i - \varepsilon_{j(i)} \right).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\widehat{\alpha}_{ATET} - \alpha_{ATET} &= \frac{1}{N_1} \sum_{D_i=1} \left(\mu_1(X_i) - \mu_0(X_i) - \alpha_{ATET} \right) \\
&+ \frac{1}{N_1} \sum_{D_i=1} \left(\varepsilon_i - \varepsilon_{j(i)} \right) \\
&+ \frac{1}{N_1} \sum_{D_i=1} \left(\mu_0(X_i) - \mu_0(X_{j(i)}) \right).
\end{aligned}
$$

Matching: Bias Problem

- We hope that we can apply a Central Limit Theorem and

$$\sqrt{N_1}(\widehat{\alpha}_{ATET} - \alpha_{ATET})$$

converges to a Normal distribution with zero mean. However,

$$E[\sqrt{N_1}(\widehat{\alpha}_{ATET} - \alpha_{ATET})] = E[\sqrt{N_1}(\mu_0(X_i) - \mu_0(X_{j(i)}))|D = 1].$$

- Now, if k is large:
  - The difference between X_i and $X_{j(i)}$ converges to zero very slowly
  - The difference $\mu_0(X_i) - \mu_0(X_{j(i)})$ converges to zero very slowly
  - $E[\sqrt{N_1}(\mu_0(X_i) - \mu_0(X_{j(i)}))|D = 1]$ may not converge to zero!
  - $E[\sqrt{N_1}(\widehat{\alpha}_{ATET} - \alpha_{ATET})]$ may not converge to zero!
- Bias is often an issue when we match in many dimensions

Matching: Solutions to the Bias Problem

The bias of the matching estimator is caused by large matching discrepancies $\|X_i - X_{j(i)}\|$. However:

1. The matching discrepancies are observed. We can always check in the data how well we are matching the covariates.
2. For $\widehat{\alpha}_{ATET}$ we can always make the matching discrepancies small by using a large reservoir of untreated units to select the matches (that is, by making N_0 large).
3. If the matching discrepancies are large, so we are worried about potential biases, we can apply bias correction techniques.
4. Partial solution: Propensity score methods (to come).

Matching with Bias Correction

- Each treated observation contributes
$$\mu_0(X_i) - \mu_0(X_{j(i)})$$
to the bias.

- Bias-corrected matching:
$$\widehat{\alpha}^{BC}_{ATET} = \frac{1}{N_1} \sum_{D_i=1} \Big((Y_i - Y_{j(i)}) - (\widehat{\mu}_0(X_i) - \widehat{\mu}_0(X_{j(i)})) \Big)$$
where $\widehat{\mu}_0(x)$ is an estimate of $E[Y|X = x, D = 0]$ (e.g., OLS).

- Under some conditions, the bias correction eliminates the bias of the matching estimator without affecting the variance.

Matching Bias: Implications for Practice

Bias arises because of the effect of large matching discrepancies on $\mu_0(X_i) - \mu_0(X_{j(i)})$. To minimize matching discrepancies:

1. Use a small M (e.g., $M = 1$). Large values of M produce large matching discrepancies.

2. Use matching with replacement. Because matching with replacement can use untreated units as a match more than once, matching with replacement produces smaller matching discrepancies than matching without replacement.

3. Try to match covariates with a large effect on $\mu_0(\cdot)$ particularly well.

Matching Estimators: Large Sample Distribution ($\widehat{\alpha}_{ATET}$)

- Matching estimators have a Normal distribution in large samples (provided that the bias is small):

$$\sqrt{N_1}(\widehat{\alpha}_{ATET} - \alpha_{ATET}) \xrightarrow{d} N(0, \sigma^2_{ATET}).$$

- For matching without replacement, the "usual" variance estimator,

$$\widehat{\sigma}^2_{ATET} = \frac{1}{N_1} \sum_{D_i=1} \left(Y_i - \frac{1}{M} \sum_{m=1}^{M} Y_{j_m(i)} - \widehat{\alpha}_{ATET} \right)^2,$$

is valid.

Matching Estimators: Large Sample Distribution ($\widehat{\alpha}_{ATET}$)

- For matching with replacement:

$$\widehat{\sigma}^2_{ATET} = \frac{1}{N_1}\sum_{D_i=1}\left(Y_i - \frac{1}{M}\sum_{m=1}^{M} Y_{j_m(i)} - \widehat{\alpha}_{ATET}\right)^2 + \frac{1}{N_1}\sum_{D_i=0}\left(\frac{K_i(K_i-1)}{M^2}\right)\widehat{\mathrm{var}}(Y_i|X_i, D_i=0),$$

  where K_i is the number of times observation i is used as a match.

- $\widehat{\mathrm{var}}(Y_i|X_i, D_i=0)$ can be computed also by matching. For example, take the observation with $D=0$ and X closest to X_i. Let $Y_{k(i)}$ be the outcome value for that observation. Then, $\mathrm{var}(Y_i|X_i, D_i=0)$ can be estimated as the sample variance of $\{Y_i, Y_{k(i)}\}$:

$$(Y_i - \bar{Y}_{ik})^2 + (Y_{k(i)} - \bar{Y}_{ik})^2,$$

  where $\bar{Y}_{ik}$ is the average of $\{Y_i, Y_{k(i)}\}$.

- The bootstrap does not work!

Propensity Score

- The **propensity score** is defined as the selection probability conditional on the confounding variables: $p(X) = P(D = 1|X)$.

- The selection on observables identification assumption is:
  1. $(Y_1, Y_0) \perp D \,|\, X$ (selection on observables)
  2. $0 < \Pr(D = 1|X) < 1$ (common support)

- Rosenbaum and Rubin (1983) proved that selection on observables implies:

$$(Y_1, Y_0) \perp D \,|\, p(X)$$

Weighting on the Propensity Score

- Weighting estimators that use the propensity score are based on the following result: If $Y_1, Y_0 \perp D|X$, then

$$\alpha_{ATE} = E\left[Y\frac{D-p(X)}{p(X)(1-p(X))}\right]$$
$$\alpha_{ATET} = \frac{1}{P(D=1)}E\left[Y\frac{D-p(X)}{1-p(X)}\right]$$

- To prove this results notice that:

$$\begin{aligned} E\left[Y\frac{D-p(X)}{p(X)(1-p(X))}\middle|X\right] &= E\left[\frac{Y}{p(X)}\middle|X,D=1\right]p(X) \\ &\quad + E\left[\frac{-Y}{1-p(X)}\middle|X,D=0\right](1-p(X)) \\ &= E[Y|X,D=1]-E[Y|X,D=0] \end{aligned}$$

- The results follow from integration over $P(X)$ and $P(X|D=1)$.

Weighting on the Propensity Score

$$\alpha_{ATE} = E\left[Y\,\frac{D-p(X)}{p(X)(1-p(X))}\right]$$
$$\alpha_{ATET} = \frac{1}{P(D=1)}E\left[Y\,\frac{D-p(X)}{1-p(X)}\right]$$

The analogy principle suggests a two step estimator:

1. Estimate the propensity score: $\widehat{p}(X)$
2. Use estimated score to produce analog estimators:

$$\widehat{\alpha}_{ATE} = \frac{1}{N}\sum_{i=1}^{N} Y_i\,\frac{D_i-\widehat{p}(X_i)}{\widehat{p}(X_i)(1-\widehat{p}(X_i))}$$
$$\widehat{\alpha}_{ATET} = \frac{1}{N_1}\sum_{i=1}^{N} Y_i\,\frac{D_i-\widehat{p}(X_i)}{1-\widehat{p}(X_i)}$$

Weighting on the Propensity Score

$$\begin{aligned}\widehat{\alpha}_{ATE} &= \frac{1}{N}\sum_{i=1}^{N} Y_i \frac{D_i - \widehat{p}(X_i)}{\widehat{p}(X_i)(1-\widehat{p}(X_i))} \\ \widehat{\alpha}_{ATET} &= \frac{1}{N_1}\sum_{i=1}^{N} Y_i \frac{D_i - \widehat{p}(X_i)}{1-\widehat{p}(X_i)}\end{aligned}$$

Standard errors:

- We need to adjust the s.e.'s for first-step estimation of $p(X)$
- Parametric first-step: Newey & McFadden (1994)
- Non-parametric first-step: Newey (1994)
- Or bootstrap the entire two-step procedure

Regression Estimators

Can we use regression estimators?

1. Least squares as an approximation

2. Least squares as a weighting scheme

3. Estimators of average treatment effects based on nonparametric regression

Least Squares as an Approximation

- $(Y_1, Y_0) \perp D|X$ implies that the conditional expectation $E[Y|D,X]$ can be interpreted as a conditional causal response function:

$$
\begin{aligned}
E[Y|D=1,X] &= E[Y_1|D=1,X] = E[Y_1|X], \\
E[Y|D=0,X] &= E[Y_0|D=0,X] = E[Y_0|X]
\end{aligned}
$$

- So $E[Y|D,X]$ provides average potential responses with and without the treatment.

- The functional form of $E[Y|D,X]$ is typically unknown, but Least Squares provides a well-defined approximation.

Least Squares as an Approximation

- Linear OLS is:

$$(\widehat{\alpha},\widehat{\beta}) = \text{argmin}_{a,b} \frac{1}{N}\sum_{i=1}^{N}\left(Y_i - aD_i - X_i'b\right)^2$$

- As a result, $(\widehat{\alpha},\widehat{\beta})$ converge to

$$(\alpha,\beta) = \text{argmin}_{a,b} E\left[\left(Y - aD - X'b\right)^2\right]$$

- It can be shown that the (α,β) that solve this minimization problem also solve:

$$(\alpha,\beta) = \text{argmin}_{a,b} E\left[\left(E[Y|D,X] - (aD + X'b)\right)^2\right]$$

- Even if $E[Y|D,X]$ is nonlinear, $\widehat{\alpha}D + X'\widehat{\beta}$ provides an estimation of a well-defined approximation to $E[Y|D,X]$.
- The result is true also for nonlinear specifications.
- But the approximation could be a bad one if the regression is severely misspecified.

Least Squares as a Weighting Scheme

- Suppose that the covariates take on a finite number of values: $x^1, x^2, \dots, x^K$. Then, from the subclassification section we know that:

$$\alpha_{ATE} = \sum_{k=1}^{K} \Big(E[Y|D=1, X=x^k] - E[Y|D=0, X=x^k] \Big) \Pr(X=x^k)$$

- Now, suppose that you run a regression that is **saturated** in X: a regression including one dummy variable, Z^k, for each possible value of X:

$$Y = \alpha D + \sum_{k=1}^{K} Z^k \beta_k + u,$$

where

$$Z^k = \begin{cases} 1 & \text{if } X = x^k \\ 0 & \text{if } X \neq x^k \end{cases}$$

Least Squares as a Weighting Scheme

- It can be shown that the coefficient $\widehat{\alpha}_{\text{OLS}}$ of the saturated regression converges to:

$$\alpha_{\text{OLS}} = \sum_{k=1}^{K} \left(E[Y|D=1, X=x^k] - E[Y|D=0, X=x^k] \right) w_k$$

where

$$w_k = \frac{\text{var}(D|X=x^k)\,\Pr(X=x^k)}{\sum_{k=1}^{K} \text{var}(D|X=x^k)\,\Pr(X=x^k)}.$$

- Strata k with a higher

$$\text{var}(D|X=x^k) = \Pr(D|X=X^k)(1-\Pr(D|X=X^k))$$

(i.e. propensity scores close to 0.5) receive higher weight. Strata with propensity scores close to 0 or 1 receive lower weights.

- OLS down-weights strata where the average causal effects are less precisely estimated.

Estimators based on Nonparametric Regression

$$\alpha_{ATE} = \int \left(E[Y|X,D=1] - E[Y|X,D=0]\right) dP(X)$$
$$\alpha_{ATET} = \int \left(E[Y|X,D=1] - E[Y|X,D=0]\right) dP(X|D=1)$$

- This suggests the following estimators:

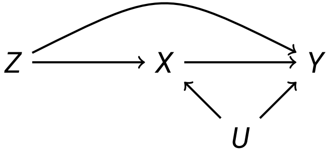
$$\widehat{\alpha}_{ATE} = \frac{1}{N}\sum_{i=1}^{N}\left(\widehat{\mu}_1(X_i) - \widehat{\mu}_0(X_i)\right)$$
$$\widehat{\alpha}_{ATET} = \frac{1}{N_1}\sum_{D_i=1}\left(\widehat{\mu}_1(X_i) - \widehat{\mu}_0(X_i)\right),$$

  where $\widehat{\mu}_1()$ and $\widehat{\mu}_0()$ are nonparametric estimators of $E[Y|X,D=1]$ and $E[Y|X,D=0]$, respectively.

- But estimating these regressions nonparametrically is difficult if the dimension of X is large (curse of dimensionality again!).

What to Match On: A Brief Introduction to DAGs

A **Directed Acyclic Graph** (**DAG**) is a set of nodes (vertices) and directed edges (arrows) with no directed cycles.



- Nodes represent variables.
- Arrows represent direct causal effects ("direct" means not mediated by other variables in the graph).
- A **causal DAG** must include:
  1. All direct causal effects among the variables in the graph
  2. All common causes (even if unmeasured) of any pair of variables in the graph

Some DAG Concepts

In the DAG:

$$
\begin{tikzcd}
Z \arrow[r] & X \arrow[r] & Y \\
& U \arrow[u] \arrow[ur] &
\end{tikzcd}
$$

- U is a **parent** of X and Y.
- X and Y are **descendants** of Z.
- There is a **directed path** from Z to Y.
- There are two **paths** from Z to U (but no directed path).
- X is a **collider** of the path $Z \to X \leftarrow U$.
- X is a **noncollider** of the path $Z \to X \to Y$.

Confounding

- Confounding arises when the treatment and the outcome have common causes.

$$\begin{tikzcd} D \arrow[rr] & & Y \\ & X \arrow[ul] \arrow[ur] & \end{tikzcd}$$

  The association between D and Y does not only reflect the causal effect of D on Y.

- Confounding creates **backdoor paths**, that is, paths starting with incoming arrows. In the DAG we can see a backdoor path from D to Y ($D \leftarrow X \rightarrow Y$).
- However, once we "block" the backdoor path by conditioning on the common cause, X, the association between D and Y is only reflective of the effect of D on Y.

$$\begin{tikzcd} D \arrow[rr] & & Y \\ & \boxed{X} \arrow[ul] \arrow[ur] & \end{tikzcd}$$

Blocked Paths

A path is blocked if and only if:

- It contains a noncollider that has been conditioned on,
- Or, it contains a collider that has not been conditioned on and has no descendants that have been conditioned on.

Examples:

1. Conditioning on a noncollider blocks a path:

$$\begin{tikzcd} \boxed{X} \arrow[r] \arrow[rr, bend left] & Z & Y \end{tikzcd}$$

2. Conditioning on a collider opens a path:

$$\begin{tikzcd} Z \arrow[r] & \boxed{X} & Y \arrow[l] \end{tikzcd}$$

3. Not conditioning on a collider (or its descendants) leaves a path blocked:

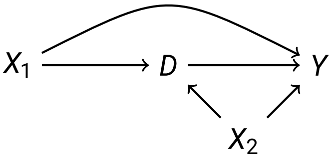
$$\begin{tikzcd} Z \arrow[r] & X & Y \arrow[l] \end{tikzcd}$$

Backdoor Criterion

- Suppose that:
  - D is a treatment,
  - Y is an outcome,
  - $X_1, \dots, X_k$ is a set of covariates.
- Is it enough to match on $X_1, \dots, X_k$ in order to estimate the causal effect of D on Y? Pearl's **Backdoor Criterion** provides sufficient conditions.
- Backdoor criterion: $X_1, \dots, X_k$ satisfies the backdoor criterion with respect to (D, Y) if:
  1. No element of $X_1, \dots, X_k$ is a descendant of D.
  2. All backdoor paths from D to Y are blocked by $X_1, \dots, X_k$.
- If $X_1, \dots, X_k$ satisfies the backdoor criterion with respect to (D, Y), then matching on $X_1, \dots, X_k$ identifies the causal effect of D on Y.

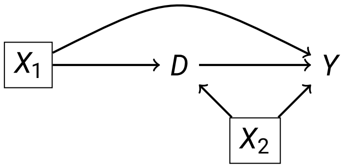
Implications for Practice

- **Matching on all common causes is sufficient:** There are two backdoor paths from D to Y.



Implications for Practice

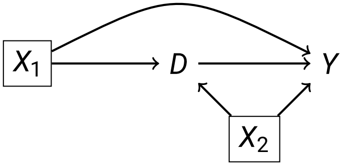
- **Matching on all common causes is sufficient:** There are two backdoor paths from D to Y.



Conditioning on X_1 and X_2 blocks the backdoor paths.

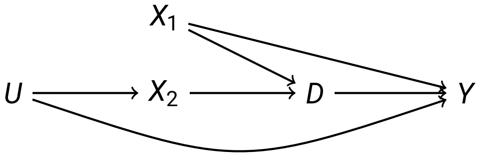
Implications for Practice

- **Matching on all common causes is sufficient:** There are two backdoor paths from D to Y.



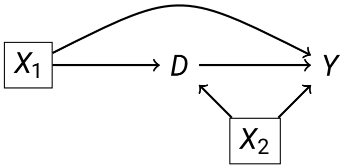
  Conditioning on X_1 and X_2 blocks the backdoor paths.

- **Matching may work even if not all common causes are observed:** U and X_1 are common causes.



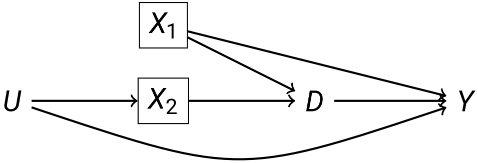
Implications for Practice

- **Matching on all common causes is sufficient:** There are two backdoor paths from D to Y.



Conditioning on X_1 and X_2 blocks the backdoor paths.

- **Matching may work even if not all common causes are observed:** U and X_1 are common causes.



Conditioning on X_1 and X_2 is enough.

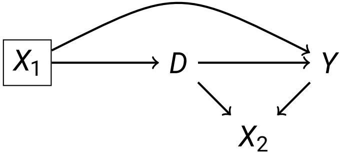
Implications for Practice (cont.)

- **Matching on an outcome may create bias:** There is only one backdoor path from D to Y.

$$\begin{tikzcd}
X_1 \arrow[rr] \arrow[rrrr, bend left] & & D \arrow[rr] \arrow[dr] & & Y \arrow[dl] \\
& & & X_2 &
\end{tikzcd}$$

Implications for Practice (cont.)

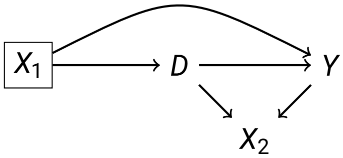
- **Matching on an outcome may create bias:** There is only one backdoor path from D to Y.



Conditioning on X_1 blocks the backdoor path. Conditioning on X_2 would open a path!

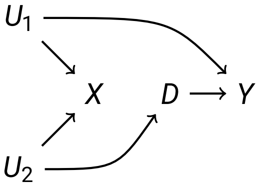
Implications for Practice (cont.)

- **Matching on an outcome may create bias:** There is only one backdoor path from D to Y.



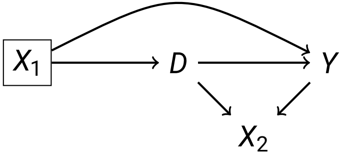
Conditioning on X_1 blocks the backdoor path. Conditioning on X_2 would open a path!

- **Matching on all pretreatment covariates is not always the answer:** There is one backdoor path and it is closed.



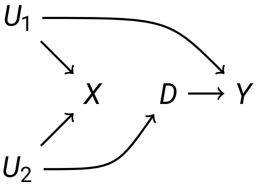
Implications for Practice (cont.)

- **Matching on an outcome may create bias:** There is only one backdoor path from D to Y.



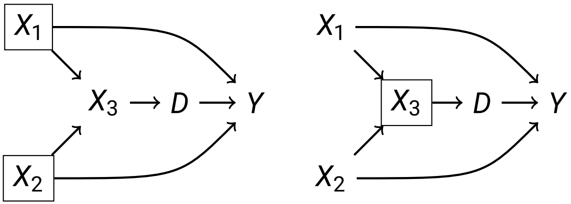
  Conditioning on X_1 blocks the backdoor path. Conditioning on X_2 would open a path!

- **Matching on all pretreatment covariates is not always the answer:** There is one backdoor path and it is closed.



Implications for Practice (cont.)

- **There may be more than one set of conditioning variables that satisfy the backdoor criterion:**



  - Conditioning on the common causes, X_1 and X_2, is sufficient, as always.
  - But conditioning on X_3 only also blocks the backdoor paths.

DAGs: Final Remarks

- The Backdoor Criterion provides a useful graphical test that can be used to select matching variables.
- The Backdoor Criterion is only a set of sufficient conditions, but covers most of the interesting cases. However, there are general results (Shpitser, VanderWeele, and Robins, 2012).
- Applying the backdoor criterion requires knowledge of the DAG. There exist results on the validity of matching under limited knowledge of the DAG (VanderWeele and Shpitser, 2011):
  - Suppose that there is a set of observed pretreatment covariates and that we know if they are causes of the treatment and/or the outcome.
  - Suppose that there exists a subset of the observed covariates such that matching on them is enough to control for confounding. Then, matching on the subset of the observed covariates that are either a cause of the treatment or a cause of the outcome or both is also enough to control for confounding.

References

Imbens, G. (2004). Nonparametric estimation of average treatment effects under exogeneity: A review. Review of Economics and Statistics, *86(1):4–29*

Pearl, J. (2000). Causality: Models, Reasoning, and Inference. *Cambridge University Press*

These slides are based on the slides by **Alberto Abadie** for previous iterations of 14.385.