

Foundations of machine learning  
Fairness and machine learning

Maximilian Kasy

Department of Economics, University of Oxford

Hilary term 2023

# Outline

- Targeted treatment assignment and supervised learning.
- Fairness as predictive parity and taste-based discrimination.
- Limitations of this notion of fairness.
- Alternative notions of fairness / discrimination.
- Social welfare as a unifying framework for many theories of justice.
- The causal impact of algorithms on inequality / social welfare.
- Case study: Predictive incarceration.

## Takeaways for this part of class

- Public debate and the computer science literature:  
**Fairness** of algorithms, understood as the absence of **discrimination**.
- We argue: Leading definitions of fairness have three limitations:
  1. They legitimize inequalities justified by “merit.”
  2. They are narrowly bracketed; only consider differences of treatment within the algorithm.
  3. They only consider between-group differences.
- Two alternative perspectives:
  1. What is the causal impact of the introduction of an algorithm on **inequality**?
  2. Who has the **power** to pick the objective function of an algorithm?

## Fairness in algorithmic decision making – Setup

- Binary treatment  $W$ , treatment return  $M$  (heterogeneous), treatment cost  $c$ .  
Decision maker's objective

$$\mu = E[W \cdot (M - c)].$$

- All expectations denote averages across individuals (not uncertainty).
- $M$  is unobserved, but predictable based on features  $X$ .  
For  $m(x) = E[M|X = x]$ , the optimal policy is

$$w^*(x) = \mathbf{1}(m(x) > c).$$

# Examples

- Bail setting for defendants based on predicted recidivism.
- Screening of job candidates based on predicted performance.
- Consumer credit based on predicted repayment.
- Screening of tenants for housing based on predicted payment risk.
- Admission to schools based on standardized tests.

Fairness and discrimination

Inequality and social welfare

Case study

References

## Definitions of fairness

- Most definitions depend on **three ingredients**.
  1. Treatment  $W$  (job, credit, incarceration, school admission).
  2. A notion of merit  $M$  (marginal product, credit default, recidivism, test performance).
  3. Protected categories  $A$  (ethnicity, gender).

- I will focus initially on the following **definition of fairness**:

$$\pi = E[M|W = 1, A = 1] - E[M|W = 1, A = 0] = 0$$

*“Average merit, among the treated, does not vary across the groups  $a$ .”*

This is called “predictive parity” in machine learning,  
the “hit rate test” for “taste based discrimination” in economics.

- “Fairness in machine learning” literature: **Constrained optimization**.

$$w^*(\cdot) = \underset{w(\cdot)}{\operatorname{argmax}} E[w(X) \cdot (m(X) - c)] \quad \text{subject to} \quad \pi = 0.$$

# Fairness and $\mathcal{D}$ 's objective

## Observation

Suppose that  $W, M$  are binary (“classification”), and that

1.  $m(X) = M$  (perfect predictability), and
2.  $w^*(x) = \mathbf{1}(m(X) > c)$  (unconstrained maximization of  $\mathcal{D}$ 's objective  $\mu$ ).

Then  $w^*(x)$  satisfies predictive parity, i.e.,  $\pi = \mathbf{0}$ .

### In words:

- If  $\mathcal{D}$  is a firm that is maximizing profits and observes everything then their decisions are fair by assumption.
  - No matter how unequal the resulting outcomes within and across groups.
- Only deviations from profit-maximization are “unfair.”



## Three normative limitations of “fairness” as predictive parity

1. They legitimize and perpetuate **inequalities justified by “merit.”**  
Where does inequality in ***M*** come from?

## Three normative limitations of “fairness” as predictive parity

1. They legitimize and perpetuate **inequalities justified by “merit.”**  
Where does inequality in  $M$  come from?
2. They are **narrowly bracketed.**  
Inequality in  $W$  in the algorithm,  
instead of some outcomes  $Y$  in a wider population.

## Three normative limitations of “fairness” as predictive parity

1. They legitimize and perpetuate **inequalities justified by “merit.”**  
Where does inequality in  $M$  come from?
2. They are **narrowly bracketed.**  
Inequality in  $W$  in the algorithm,  
instead of some outcomes  $Y$  in a wider population.
3. Fairness-based perspectives **focus on categories** (protected groups)  
and ignore within-group inequality.

## Alternative measures of fairness (1)

Measures that share the same limitations:

- Equality of true positives:

$$E[W|M = 1, A = 1] - E[W|M = 1, A = 0].$$

- Equality of false positives:

$$E[W|M = 0, A = 1] - E[W|M = 0, A = 0].$$

- Balance for the negative class:

$$E[M|W = 0, A = 1] - E[M|W = 0, A = 0]$$

(Like predictive parity, but for  $W = 0$ .)

## Alternative measures of fairness (2)

Measures which share only some of these limitations:

- Disparate impact and demographic parity:

$$\frac{E[W|A = 1]}{E[W|A = 0]}, \quad E[W|A = 1] - E[W|A = 0].$$

- Conditional statistical parity:

$$E[W|A = 1, X' = x'] - E[W|A = 0, X' = x']$$

for a subset of features  $X'$  considered “legitimate” sources of inequality.  
(Cf. Oaxaca-Blinder decompositions.)

- Individual fairness:

$$E[W|X = x_i] - E[W|X = x_j] \text{ for } d(i,j) \approx 0,$$

for a measure of distance  $d(i,j)$  between individuals.

## Practice problem

- Which of these measures of fairness do you find more or less appealing?
- Why? For which contexts or applications?

Fairness and discrimination

Inequality and social welfare

Case study

References

# Social welfare

- The framework of fairness / bias / discrimination contrasts with perspectives focused on *consequences for social welfare*.
- Common presumption for most theories of justice:

Normative statements about society  
are based on statements about individual welfare

- Formally:
  - Individuals  $i = 1, \dots, n$
  - Individual  $i$ 's welfare  $Y_i$
  - Social welfare as function of individuals' welfare

$$SWF = F(Y_1, \dots, Y_n).$$



## Practice problem

- **Who is to be included** among  $i = 1, \dots, n$ ?
  - All citizens? All residents? All humans on earth?
  - Future generations? Animals?
- **How to measure individual welfare**  $Y_i$ ?
  - Opportunities or outcomes?
  - Utility? Resources? Capabilities?
- **How to aggregate** to *SWF*?  
How much do we care about
  - Trevon vs. Emily, Sophie vs. José?
  - Millionaires vs. homeless people?
  - Sick vs. healthy people?
  - Groups that were victims of historic injustice?

## The impact on inequality or welfare as an alternative to fairness

- Outcomes are determined by the **potential outcome equation**

$$Y = W \cdot Y^1 + (1 - W) \cdot Y^0.$$

- The **realized outcome** distribution is given by

$$p_{Y,X}(y,x) = \left[ p_{Y^0|X}(y,x) + w(x) \cdot \left( p_{Y^1|X}(y,x) - p_{Y^0|X}(y,x) \right) \right] \cdot p_X(x).$$

- What is the impact of  $w(\cdot)$  on a **statistic**  $v$ ?

$$v = v(p_{Y,X}).$$

Examples: Variance, quantiles, between group inequality.

- Cf. Distributional decompositions in labor economics!

# When fairness and equality are in conflict

- Fairness is about **treating** people of the same “**merit**” independently of their **group** membership.
- Equality is about the (counterfactual / causal) **consequences** of an algorithm for the distribution of **welfare** of different **people**.

Examples when they are in conflict:

1. Increased surveillance / **better prediction** algorithms:  
Lead to treatments more aligned with “merit”  
Good for fairness, bad for equality.
2. Affirmative action / **compensatory interventions** for pre-existing inequalities:  
Bad for fairness, good for equality.

## Influence function approximation of the statistic $\mathbf{v}$

$$\mathbf{v}(\mathbf{p}_{Y,X}) - \mathbf{v}(\mathbf{p}_{Y,X}^*) = E[IF(Y, X)] + o(\|\mathbf{p}_{Y,X} - \mathbf{p}_{Y,X}^*\|).$$

- $IF(Y, X)$  is the influence function of  $\mathbf{v}(\mathbf{p}_{Y,X})$ .

Formally: The Riesz representer of the Fréchet derivative of  $\mathbf{v}$ .

- The expectation averages over the distribution  $\mathbf{p}_{Y,X}$ .

# The impact of marginal policy changes on profits, fairness, and inequality

## Proposition

Consider a family of assignment policies  $w(\mathbf{x}) = w^*(\mathbf{x}) + \varepsilon \cdot dw(\mathbf{x})$ . Then

$$\partial_\varepsilon \mu = E[dw(X) \cdot l(X)], \quad \partial_\varepsilon \pi = E[dw(X) \cdot p(X)], \quad \partial_\varepsilon v = E[dw(X) \cdot n(X)],$$

# The impact of marginal policy changes on profits, fairness, and inequality

## Proposition

Consider a family of assignment policies  $w(x) = w^*(x) + \varepsilon \cdot dw(x)$ . Then

$$\partial_\varepsilon \mu = E[dw(X) \cdot l(X)], \quad \partial_\varepsilon \pi = E[dw(X) \cdot p(X)], \quad \partial_\varepsilon v = E[dw(X) \cdot n(X)],$$

where

$$\begin{aligned} l(X) &= E[M|X=x] - c, \\ p(X) &= E \left[ (M - E[M|W=1, A=1]) \cdot \frac{A}{E[WA]} \right. \\ &\quad \left. - (M - E[M|W=1, A=0]) \cdot \frac{(1-A)}{E[W(1-A)]} \middle| X=x \right], \\ n(x) &= E \left[ IF(Y^1, x) - IF(Y^0, x) \middle| X=x \right]. \end{aligned}$$

# Uses of the proposition

1. Elucidate the **tension** between objectives.
  - Profits vs. fairness vs. equality vs. welfare?
  - Suppose  $\pi < 0$ ,  $n(\mathbf{x}) > 0$  is positive, while  $p(\mathbf{x}) < 0$ .  
Then increasing  $w(\mathbf{x})$  is good for welfare and bad for fairness.
  - $\Rightarrow$  Characterizes which parts of the feature space drive the tension between alternative objectives.

# Uses of the proposition

1. Elucidate the **tension** between objectives.
  - Profits vs. fairness vs. equality vs. welfare?
  - Suppose  $\pi < 0$ ,  $n(x) > 0$  is positive, while  $p(x) < 0$ .  
Then increasing  $w(x)$  is good for welfare and bad for fairness.
  - $\Rightarrow$  Characterizes which parts of the feature space drive the tension between alternative objectives.
2. Solve for **optimal assignment** subject to constraints.
  - E.g. maximize  $\mu$  subject to  $\pi = 0$ .
  - Then  $w(x) = \mathbf{1}(l(x) > \lambda p(x))$ .



# Uses of the proposition 1, continued

## 3. **Power and inverse welfare weights**

- For a given  $w(\cdot)$ , what objective is implicitly maximized?
- What are the weights for different individuals that rationalize  $w(\cdot)$ ?

# Uses of the proposition 1, continued

## 3. **Power and inverse welfare weights**

- For a given  $w(\cdot)$ , what objective is implicitly maximized?
- What are the weights for different individuals that rationalize  $w(\cdot)$ ?

## 4. **Algorithmic auditing.**

- Similar to distributional decompositions in labor economics.
- Cf. Fortin and Lemieux (1997); Firpo et al. (2009).

# Power

- Both fairness and equality are about differences between people who are **being treated**.
- Elephant in the room:
  - Who is on the **other side** of the algorithm?
  - Who gets to be the decision maker  $\mathcal{D}$  – who gets to pick the objective function  $\mu$ ?
- Political economy perspective:
  - **Ownership of the means of prediction.**
  - Data and algorithms.

Fairness and discrimination

Inequality and social welfare

Case study

References

## Case study

- Compas risk score data for recidivism.
- From Pro-Publica's reporting on algorithmic discrimination in sentencing.

Mapping our setup to these data:

- **A**: race (Black or White),
- **W**: risk score exceeding 4,
- **M**: recidivism within two years,
- **Y**: jail time,
- **X**: race, sex, age, juvenile counts of misdemeanors, felonies, and other infractions, general prior counts, as well as charge degree.

## Counterfactual scenarios

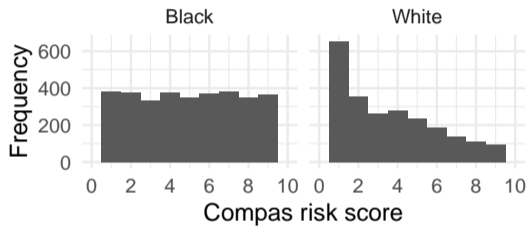
Compare three scenarios:

1. "Affirmative action:" Adjust risk scores  $\pm 1$ , depending on race.
2. Status quo.
3. Perfect predictability: Scores equal **10** or **1**, depending on recidivism in 2 years.

For each: Impute counterfactual

- **W**: Counterfactual score bigger than 4.
- **Y**: Based on a causal-forest estimate of the impact on **Y** of risk scores, conditional on the covariates in **X**.
- This relies on the assumption of conditional exogeneity of risk-scores given **X**. Not credible, but useful for illustration.

## Compas risk scores



## Estimated effect of scores

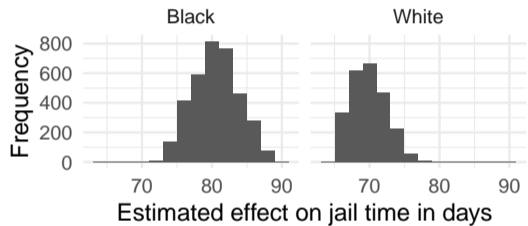


Table: Counterfactual scenarios, by group

Scenario	Black			White		
	(Score>4)	Recid (Score>4)	Jail time	(Score>4)	Recid (Score>4)	Jail time
Aff. Action	0.49	0.67	49.12	0.47	0.55	36.90
Status quo	0.59	0.64	52.97	0.35	0.60	29.47
Perfect predict.	0.52	1.00	65.86	0.40	1.00	42.85

Table: Counterfactual scenarios, outcomes for all

Scenario	Score>4	Jail time	IQR jail time	SD log jail time
Aff. Action	0.48	44.23	23.8	1.81
Status quo	0.49	43.56	25.0	1.89
Perfect predict.	0.48	56.65	59.9	2.10



## References

- *Pessach, D. and Shmueli, E. (2020). Algorithmic fairness. arXiv preprint arXiv:2001.09784*
- *Kasy, M. and Abebe, R. (2021). Fairness, equality, and power in algorithmic decision making. ACM Conference on Fairness, Accountability, and Transparency.*
- *Kasy, M. (2016). Empirical research on economic inequality. <http://inequalityresearch.net/>*
- *Roemer, J. E. (1998). Theories of distributive justice. Harvard University Press, Cambridge.*