Problemset 1, Econ 980w, Spring 2019: Estimating the Pareto parameter

1 By hand exercises

1. Suppose a data-set contains the following observations on the wealth of 10 random rich individuals (in thousands):
   $100, 105, 112, 120, 129, 141, 158, 183, 224, 316.$
   Calculate an estimate of the Pareto parameter α, and of $E[Y|Y > 100]$.

2. Now suppose that you just observe that the number of people in the tax bracket $[100, 200]$ equals 800, and the number of people with wealth above 200 equals 200.
   Calculate an estimate of the Pareto parameter α, and of $E[Y|Y > 100]$.

2 R exercises

Write code that performs the following:

1. Generate n independent draws from the Pareto distribution with parameters $\underline{y}$ and α

   Hint: You can take $Y_i = \underline{y} \cdot U_i^{-1/\alpha}$ for U uniformly $[0, 1]$ distributed. Why?

2. Save these data to a .csv file, and exchange your file with a classmate.

3. Use your classmate's data to estimate α, using the formula
$$\widehat{\alpha}^{MLE} = \frac{n}{\sum_i \log\left(y_i/\underline{y}\right)}.$$

4. Now generate new data using the same procedure as before, and just tell your classmate the value $\underline{y}$, as well as the number of observation below / above the cutoff $2 \cdot \underline{y}$. Ask her/him to provide an estimate of α based on these numbers, using
$$\widehat{\alpha}^{MLE} = \frac{\log(N_2/n)}{\log\left(\underline{y}/y_l\right)}.$$

5. Now we are going to verify the result underpinning Piketty's argument about "$r - g$" by simulations. Generate data following the process

$$Y_{t+1} = w_t + R_t \cdot Y_t,$$

where w_t and R_t are independent draws from uniform distributions with boundary values that you pick. Generate 10.000 observations, and only keep the last 2.000. Save them, and give them to a classmate.

6. Sort the data you got from your classmate, and only keep the top 200. Use these observations to estimate the Pareto parameter as in step 2.

7. Repeat the last two steps, but for a different distribution of R_t. Does the estimated Pareto parameter change in the way that you would expect?