Who wins, who loses?
Tools for distributional policy evaluation

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- Few policy changes result in Pareto improvements
- Most generate WINNERS and LOSERS

EXAMPLES:

1. **Trade liberalization**
   net producers vs. net consumers of goods with rising / declining prices
2. **Progressive income tax reform**
   high vs. low income earners
3. **Price change of publicly provided good** (health, education,...)
   Inframarginal, marginal, and non-consumers of the good; tax-payers
4. **Migration**
   Migrants themselves; suppliers of substitutes vs. complements to migrant labor
5. **Skill biased technical change**
   suppliers of substitutes vs. complements to technology; consumers

This implies...

If we evaluate social welfare based on individuals' welfare:

1. To **evaluate** a **policy** effect, we need to
   1.1 define how we measure individual gains and losses,
   1.2 estimate them, and
   1.3 take a stance on how to aggregate them.
2. To **understand political economy**, we need to characterize the sets of winners and losers of a policy change.

My objective:

1. tools for distributional evaluation
2. utility-based framework, arbitrary heterogeneity, endogenous prices

Proposed procedure

1. impute money-metric welfare effect to each individual
2. then:
   2.1 report average effects given income / other covariates
   2.2 construct sets of winners and losers (in expectation)
   2.3 aggregate using welfare weights

contrast with program evaluation approach:

1. effect on average
2. of observed outcome

Contributions

1. **Assumptions**
    1.1 endogenous prices / wages (vs. public finance)
    1.2 utility-based social welfare (vs. labor, distributional decompositions)
    1.3 arbitrary heterogeneity (vs. labor)
2. **Objects of interest**
    2.1 disaggregated welfare ⇒
        - political economy
        - allow reader to have own welfare weights
    2.2 aggregated ⇒ policy evaluation as in optimal taxation
3. **Formal results**
    next slide

Formal results

1. **Identification**
   1.1 Main challenge: $E[\dot{w} \cdot l | w \cdot l, \alpha]$
   1.2 More generally: $E[\dot{x}|x, \alpha]$ causal effect of policy *conditional* on endogenous outcomes,
   1.3 solution: tools from vector analysis, fluid dynamics
2. **Aggregation**
   social welfare & distributional decompositions
   2.1 welfare weights $\approx$ derivative of influence function
   2.2 welfare impact $=$ impact on income - behavioral correction
3. **Inference**
   3.1 local linear quantile regressions
   3.2 combined with control functions
   3.3 suitable weighted averages

▸ Literature

Abbring and Heckman (2007)	**this paper**
Distribution of treatment effects for a discrete treatment $F(\Delta Y \| X)$	Conditional expectation of marginal causal effect of continuous treatment given outcome $E[\partial_X Y \| Y, X]$
prediction of GE effects for counterfactual policy	ex-post evaluation of realized price/wage changes
effect on realized outcomes, ΔY	equivalent variation $l \cdot \dot{w}$

Notation

- **policy $\alpha \in \mathbb{R}$**
  individuals i
- potential outcome w^α
  realized outcome w
- partial derivatives $\partial_w := \partial / \partial w$
  with respect to policy $\dot{w} := \partial_\alpha w^\alpha$
- density f
  cdf F
  quantile Q
- wage w
  labor supply l
  consumption vector c
  taxes t
  covariates W

Setup

Assumption (Individual utility maximization)

individuals choose c and l to solve

$$\max_{c,l} u(c,l) \quad s.t. \quad c \cdot p \leq l \cdot w - t(l \cdot w) + y_0. \tag{1}$$

$$v := \max u$$

- *u, c, l, w vary arbitrarily across i*
- p, w, y_0, t *depend on* α
  $\Rightarrow$ *so do c, l, and v*
- *u differentiable, increasing in c, decreasing in l, quasiconcave, does not depend on* α

Objects of interest

Definition

1. *Money metric utility impact of policy:*

$$\dot{e} := \dot{v} \big/ \partial_{y_0} v$$

2. *Average conditional policy effect on welfare:*

$$\gamma(y, W) := E[\dot{e} | y, W, \alpha]$$

3. *Sets of winners and losers:*

$$\mathscr{W} := \{(y, W) : \gamma(y, W) \geq 0\}$$
$$\mathscr{L} := \{(y, W) : \gamma(y, W) \leq 0\}$$

4. *Policy effect on social welfare: $SWF : \; v(.) \to \mathbb{R}$*

$$S\dot{W}F = E[\omega \cdot \gamma]$$

Marginal policy effect on individuals

Lemma

$$\begin{aligned} \dot{y} &= (\dot{l} \cdot w + l \cdot \dot{w}) \cdot (1 - \partial_z t) - \dot{t} + \dot{y}_0, \\ \dot{e} &= \qquad\quad l \cdot \dot{w} \cdot (1 - \partial_z t) - \dot{t} + \dot{y}_0 - c \cdot \dot{p}. \end{aligned} \tag{2}$$

Proof: Envelope theorem.

1. wage effect $l \cdot \dot{w} \cdot (1 - \partial_z t)$,
2. effect on unearned income $\dot{y}_0$,
3. mechanical effect of changing taxes $-\dot{t}$.
4. behavioral effect $b := \dot{l} \cdot w \cdot (1 - \partial_z t) = \dot{l} \cdot n$,
5. price effect $-c \cdot \dot{p}$.

Income vs utility:

$$\dot{y} - \dot{e} = \dot{l} \cdot n + c \cdot \dot{p}.$$

Example: Introduction of EITC (cf. Rothstein, 2010)

- Transfer income to poor mothers made contingent on labor income
  1. mechanical effect > 0 if employed
     < 0 if unemployed
  2. labor supply effect > 0
  3. wage effect < 0

     for mothers *and* non-mothers
- Evaluation based on
  1. **income** (“labor”)
  2. **utility**, assuming **fixed wages** (“public”)
  3. **utility, general** model
-
  1. mechanical + wage + labor supply
  2. mechanical
  3. mechanical + wage
- Case 3 looks worse than “labor” / “public” evaluations

Identification of disaggregated welfare effects

- Goal: **identify** $\gamma(\mathbf{y},\mathbf{W}) = \mathbf{E}[\dot{\mathbf{e}}|\mathbf{y},\mathbf{W},\alpha]$
- Simplified case:
  no change in prices, taxes, unearned income
  no covariates
- Then

$$\gamma(y) = E[l \cdot (1 - \partial_z t) \cdot \dot{w} | l \cdot w, \alpha]$$

- Denote $x = (l, w)$.
  Need to identify

$$g(x,\alpha) = E[\dot{x}|x,\alpha] \tag{3}$$

  from

$$f(x|\alpha).$$

- Made necessary by combination of
  1. utility-based social welfare
  2. heterogeneous wage response.

Assume :

1. $x = x(\alpha, \varepsilon)$, $x \in \mathbb{R}^k$
2. $\alpha \perp \varepsilon$
3. $x(.,\varepsilon)$ differentiable

Physics analogy:

- $x(\alpha, \varepsilon)$: position of particle ε at time α
- $f(x|\alpha)$: density of gas / fluid at time α, position x
- $\dot{f}$ change of density
- $h(x, \alpha) = E[\dot{x}|x, \alpha] \cdot f(x|\alpha)$: "flow density"

Stirring your coffee

- If we know densities $f(x|\alpha)$,
- what do we know about flow $g(x,\alpha) = E[\dot{x}|x,\alpha]$?

Problem: Stirring your coffee

- does not change its density,
- yet moves it around.
- $\Rightarrow$ different flows $g(x,\alpha)$ consistent with a constant density $f(x|\alpha)$

Will show:

- Knowledge of $f(x|\alpha)$
  - identifies $\nabla \cdot h = \sum_{j=1}^{k} \partial_{x^j} h^j$
  - where $h = E[\dot{x}|x,\alpha] \cdot f(x|\alpha)$,
  - identifies nothing else.
- Add to h
  - $\tilde{h}$ such that $\nabla \cdot \tilde{h} \equiv 0$
  - $\Rightarrow f(x|\alpha)$ does not change
  - “stirring your coffee”
- Additional conditions
  - e.g.: “wage response unrelated to initial labor supply”
  - $\Rightarrow$ just-identification of $g(x,\alpha) = E[\dot{x}|x,\alpha]$
  - $g^j(x,\alpha) = \partial_\alpha Q(v^j|v^1,\ldots,v^{j-1},.\alpha)$

Density and flow

Recall

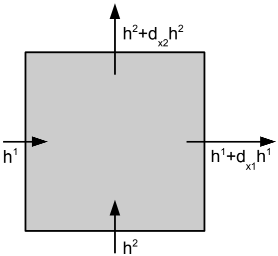
$$h(x,\alpha) := E[\dot{x}|x,\alpha]\cdot f(x|\alpha)$$

$$\nabla \cdot h := \sum_{j=1}^{k} \partial_{x^j} h^j$$

$$\dot{f} := \partial_\alpha f(x|\alpha)$$

Theorem

$$\dot{\mathbf{f}} = -\nabla \cdot \mathbf{h} \tag{4}$$



Proof:

1. For some $a(x)$, let

$$A(\alpha) := E[a(x(\alpha,\varepsilon))|\alpha] = \int a(x(\alpha,\varepsilon))dP(\varepsilon)$$
$$= \int a(x)f(x|\alpha)dx.$$

2. By partial integration:

$$\dot{A}(\alpha) = E[\partial_x a \cdot \dot{x}|\alpha] = \sum_{j=1}^{k} \int \partial_{x^j} a \cdot h^j dx$$
$$= -\int a \cdot \sum_{j=1}^{k} \partial_{x^j} h^j dx = -\int a \cdot (\nabla \cdot h) dx.$$

3. Alternatively:

$$\dot{A}(\alpha) = \int a(x)\dot{f}(x|\alpha)dx.$$

4. 2 and 3 hold for any $a \Rightarrow \dot{f} = -\nabla \cdot h$. □

The identified set

Theorem

The identified set for h is given by

$$h^0 + \mathscr{H} \tag{5}$$

where

$$\mathscr{H} = \{\tilde{h} : \nabla \cdot \tilde{h} \equiv 0\}$$
$$h^{0j}(x,\alpha) = f(x|\alpha) \cdot \partial_\alpha Q(v^j | v^1, \ldots, v^{j-1}, \alpha)$$
$$v^j = F(x^j | x^1, \ldots, x^{j-1}, \alpha)$$

▸ Proof

Theorem

1. *Suppose $k = 1$. Then*

$$\mathscr{H} = \{\tilde{h} \equiv 0\}. \tag{6}$$

2. *Suppose $k = 2$. Then*

$$\mathscr{H} = \{\tilde{h} : \ \tilde{h} = A \cdot \nabla H \textit{ for some } H : \mathscr{X} \to \mathbb{R}\}. \tag{7}$$

where

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

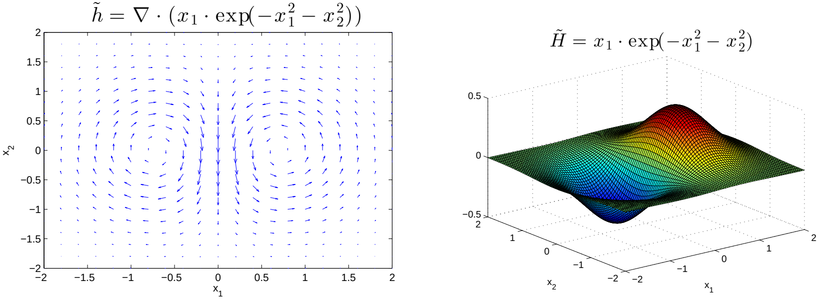
3. *Suppose $k = 3$. Then*

$$\mathscr{H} = \{\tilde{h} : \ \tilde{h} = \nabla \times G\}. \tag{8}$$

where $G : \mathscr{X} \to \mathbb{R}^3$.

Proof: Poincaré's Lemma.

Figure : Incompressible flow and rotated gradient of potential



Point identification

Theorem

Assume

$$\frac{\partial}{\partial x^j} E[\dot{x}^i | x, \alpha] = 0 \; for \; j > i. \tag{9}$$

Then h is point identified, and equal to h^0 as defined before.

In particular

$$\begin{aligned} g^j(x,\alpha) &= E[\dot{x}^j | x, \alpha] \\ &= \partial_\alpha Q(v^j | v^1, \ldots, v^{j-1}, \alpha). \end{aligned}$$

▸ Identification with controls, identification of γ

Aggregation

- Relationship
  social welfare $\Leftrightarrow$ distributional decompositions?
- public finance welfare weights
  $\approx$ derivative of dist decomp influence functions

  Theorem: Welfare weights and influence functions
- Alternative representations of $S\dot{W}F$
  $\Rightarrow$ alternative ways to estimate $S\dot{W}F$:
  1. weighted average of individual welfare effects $\dot{e}$, γ
  2. distributional decomposition for counterfactual income $\tilde{y}$
     (holding labor supply constant)
  3. distributional decomposition of realized income
     minus behavioral correction

  Theorem: Alternative representations

Estimation

1. First estimate the disaggregated welfare impact

$$\begin{aligned}\gamma(y,W) &= E[\dot{e}|y,W,\alpha] \\ &= E[l \cdot \dot{w} \cdot (1-\partial_z t) - \dot{t} + \dot{y}_0 - c \cdot \dot{p}|y,W,\alpha] \qquad (10)\end{aligned}$$

▸ Estimation of g and γ

2. Then estimate other objects by plugging in $\widehat{\gamma}$:

$$\begin{aligned}\widehat{\mathscr{W}} &= \{(y,W) : \widehat{\gamma}(y,W) \geq 0\} \\ \widehat{\mathscr{L}} &= \{(y,W) : \widehat{\gamma}(y,W) \leq 0\} \\ \widehat{S\dot{W}F} &= E_N[\omega_i \cdot \widehat{\gamma}(y_i,W_i)]. \qquad (11)\end{aligned}$$

3. ▸ Inference

Application: distributional impact of EITC

- Following Leigh (2010)
  (see also Meyer and Rosenbaum (2001), Rothstein (2010))
- CPS-MORG
- Variation in state top-ups of EITC
  across time and states
- $\alpha =$ maximum EITC benefit available
  (weighted average across family sizes)

State EITC supplements 1984-2002

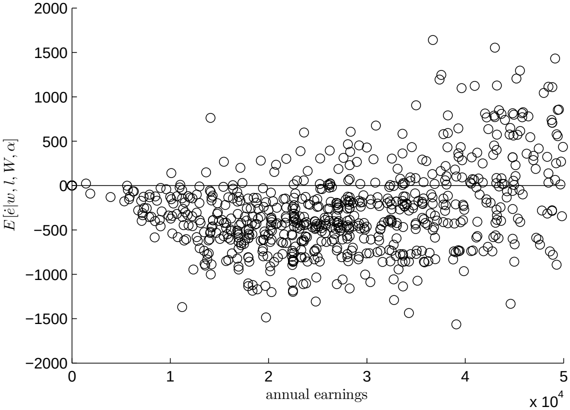
State:	CO	DC	IA	IL	KS	MA	MD	ME	MN	MN	NJ	NY	OK	OR	RI	VT	WI	WI	WI
# chld.							1+		0	1+		1+					1	2	3+
1984																	30	30	30
1985																	30	30	30
1986															22.21				
1987															23.46				
1988															22.96	23			
1989															22.96	25	5	25	75
1990			5												22.96	28	5	25	75
1991			6.5						10	10					27.5	28	5	25	75
1992			6.5						10	10					27.5	28	5	25	75
1993			6.5						15	15					27.5	28	5	25	75
1994			6.5						15	15		7.5			27.5	25	4.4	20.8	62.5
1995			6.5						15	15		10			27.5	25	4	16	50
1996			6.5						15	15		20			27.5	25	4	14	43
1997			6.5			10			15	15		20		5	27.5	25	4	14	43
1998			6.5		10	10	10		15	25		20		5	27	25	4	14	43
1999	8.5		6.5		10	10	10		25	25		20		5	26.5	25	4	14	43
2000	10	10	6.5	5	10	10	15	5	25	25	10	22.5		5	26	32	4	14	43
2001	10	25	6.5	5	10	15	16	5	33	33	15	25		5	25.5	32	4	14	43
2002	0	25	6.5	5	15	15	16	5	33	33	17.5	27.5	5	5	25	32	4	14	43

Leigh (2010)

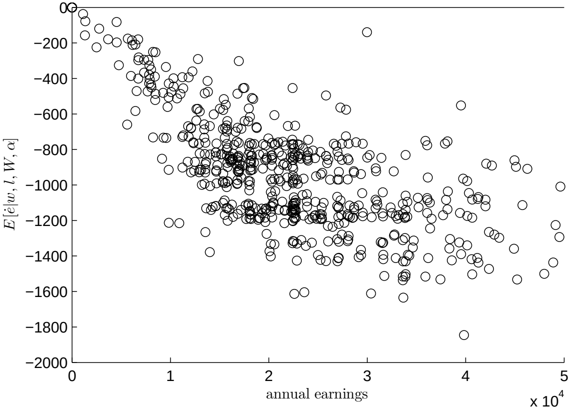
	All adults	**High school dropouts**	**High school diploma only**	**College graduates**
	dependent variable: Log real hourly wage			
Log maximum EITC	-0.121	-0.488	-0.221	0.008
	[0.064]	[0.128]	[0.073]	[0.056]
Fraction EITC-eligible	9%	25%	12%	3%
	dependent variable: whether employed			
Log maximum EITC	0.033	0.09	0.042	0.008
	[0.012]	[0.046]	[0.019]	[0.022]
Fraction EITC-eligible	14%	34%	17%	4%

Welfare effects of wage changes induced by a 10% expansion of the EITC

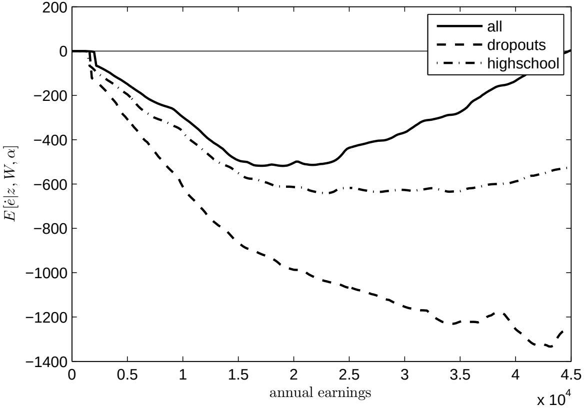
estimated welfare effect $l \cdot \dot{w}$ for a subsample of 1000 households, plotted against their earnings.



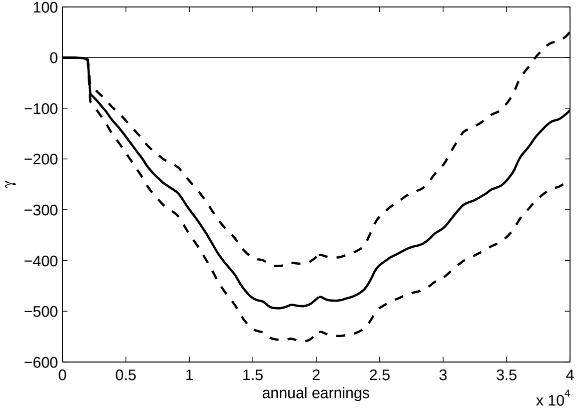
Welfare effects of wage changes induced by a 10% expansion of the EITC, high school dropouts



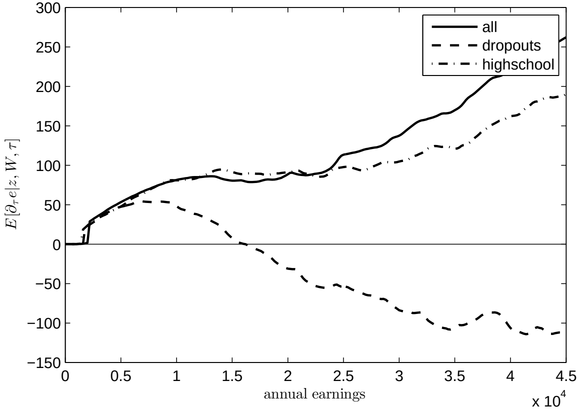
Kernel regression of welfare effects on earnings



95% confidence band for welfare effects given earnings



For comparison: Welfare effects of wage changes over the period 1989-2002



Conclusion and Outlook

- Most policies generate winners and losers
- Motivates interest in
  1. disaggregated welfare effects
  2. sets of winners and losers (political economy!)
  3. weighted average welfare effects (optimal policy!)
- Consider framework which allows for
  1. endogenous prices / wages (vs. public finance)
  2. utility-based social welfare (vs. labor, distributional decompositions)
  3. arbitrary heterogeneity (vs. labor)

Main results

1. **Identification**
   1.1 Main challenge: $E[\dot{w} \cdot l | w \cdot l, \alpha]$
   1.2 More generally: $E[\dot{x} | x, \alpha]$ causal effect of policy *conditional* on endogenous outcomes,
   1.3 solution: tools from vector analysis, fluid dynamics

Generalization to dim$(\alpha) > 1$

2. **Aggregation**
   social welfare & distributional decompositions
   2.1 welfare weights $\approx$ derivative of influence function
   2.2 welfare impact $=$ impact on income - behavioral correction
3. **Inference**
   3.1 local linear quantile regressions
   3.2 combined with control functions
   3.3 suitable weighted averages

Thanks for your time!

Literature

1. **public - optimal taxation**
   Samuelson (1947), Mirrlees (1971), Saez (2001), Chetty (2009), Hendren (2013), Saez and Stantcheva (2013)
2. **labor - determinants of wage distribution**
   Autor et al. (2008), Card (2009)
3. **distributional decompositions**
   Oaxaca (1973), DiNardo et al. (1996), Firpo et al. (2009), Rothe (2010), Chernozhukov et al. (2013)
4. **sociology - class analysis**
   Wright (2005)
5. **mathematical physics - fluid dynamics, differential forms**
   Rudin (1991)
6. **econometrics - various**
   Koenker (2005), Hoderlein and Mammen (2007), Abbring and Heckman (2007), Matzkin (2003), Altonji and Matzkin (2005)

Back

Proof of sharpness of identified set:

1. For any h s.t. $\dot{f} = -\nabla \cdot h$ construct DGP as follows
2. Let $\varepsilon = x(0,\varepsilon)$, $f(\varepsilon) = f(x|\alpha = 0)$
3. Let $x(.,\varepsilon)$ be the solution of the ODE

$$\dot{x} = g(x,\alpha), \; x(0,\varepsilon) = \varepsilon.$$

   (existence: Peano's theorem)
4. $\Rightarrow$ consistent with h and with f

□

▸ Back

Controls; back to γ

Proposition

- *Suppose* $\alpha \perp \varepsilon | \mathbf{W}$, *and* $\frac{\partial}{\partial x^j} E[\dot{x}^i | x, W, \alpha] = 0$ *for* $j > i$. *Then*

$$E[\dot{x}^j | x, W, \alpha] = \partial_\alpha Q(v^j | v^1, \ldots, v^{j-1}, W, \alpha),$$
$$\textit{where } v^j = F(x^j | x^1, \ldots, x^{j-1}, W, \alpha).$$

- *If* $x^j = n$,

$$\gamma(\mathbf{y}, \mathbf{W}) = E[l \cdot \dot{n} | y, W, \alpha] = \mathbf{E}[\mathbf{l} \cdot \partial_\alpha \mathbf{Q}(v^j | v^1, \ldots, v^{j-1}, W, \alpha) | \mathbf{y}, \mathbf{W}, \alpha]. \quad (12)$$

panel data, instrumental variables: similar (see paper)

▸ Back

Welfare weights and influence functions

Consider $\theta : P^y \to \mathbb{R}$

Theorem

1. ***Welfare weights:***

$$\dot{SWF} = E[\omega^{SWF} \cdot \dot{e}] \quad (13)$$

$$\dot{\theta} = E[\omega^{\theta} \cdot \dot{y}].$$

2. ***Influence function:***

$$\dot{\theta} = \partial_\alpha E\left[IF(y^\alpha)\right] = \partial_\alpha \int IF(y) dF_{y^\alpha}(y).$$

3. ***Relating the two:***

$$\omega^{\theta} = \partial_y IF(y).$$

▸ Back

Alternative representations

Theorem

- *Assume* $\omega^{SWF} = \omega^{\theta} = \omega$ *and* $\dot{p} = 0$.
- *Let*

$$\tilde{y}^{\alpha} = l^0 \cdot w^{\alpha} - t^{\alpha}(l^0 \cdot w^{\alpha}) + y_0^{\alpha},$$

$$b = \dot{l} \cdot n$$

Then $\dot{e} = \dot{\tilde{y}} = \dot{y} - b$ *and*

1. ***Counterfactual income distribution:***

$$\begin{aligned} \dot{\mathbf{SWF}} &= E[\omega \cdot \dot{\tilde{y}}] = \mathbf{E}[\omega \cdot \gamma] \\ &= \partial_{\alpha} \theta \left(P^{\tilde{y}^{\alpha}} \right) \\ &= \partial_{\alpha} E\left[IF(\tilde{y}^{\alpha}) \right]. \end{aligned} \tag{14}$$

2. ***Behavioral correction of distributional decomposition:***

$$\dot{\theta} - \dot{SWF} = E[\omega \cdot b]. \tag{15}$$

Back

Estimation of g and γ

1) $\widehat{v}^j$: estimate of $F(x^j|x^1,\ldots,x^{j-1},W,\alpha)$

1. estimated **conditional quantile** of x^j given $(W,\alpha,\widehat{v}^1,\ldots,\widehat{v}^{j-1})$
2. estimate by local average
3. **local weights:** K_i^j for observation i around $(W,\alpha,\widehat{v}^1,\ldots,\widehat{v}^{j-1})$
4. $$\widehat{v}^j = \frac{E_N[K_i^j \cdot \mathbf{1}(x_i^j \leq x^j)]}{E_N[K_i^j]} \tag{16}$$

Back

2) $\widehat{g}^j$: estimate of $E[\dot{x}^j|x, W\alpha]$

1. identified by slope of **quantile regression**
2. estimate by local linear Qreg
3. **regression residual:** $U_i^j = x_i^j - x^j - g \cdot \alpha_i$
4. **loss function:** $L_i^j = U_i^j \cdot (\widehat{v}^j - \mathbf{1}(U_i^j \leq 0))$
5. $$\widehat{g}^j = \underset{g}{\text{argmin}}\ E_N\left[K_i^j \cdot L_i^j\right],$$

3) $\widehat{\gamma}(y, W)$: estimate of $E[I \cdot \dot{n}|y, W, \alpha]$

1. $n = x^j \Rightarrow \dot{e} = I \cdot \dot{n} = I \cdot \dot{x}^j$
2. estimate γ by **weighted average**

$$\widehat{\gamma}(y, W) = \frac{E_N\left[K_i \cdot I \cdot \widehat{g}^j\right]}{E_N[K_i]}$$

Inference

- $\widehat{g}^j$ depends on data in 3 ways:
  1. through x^j, α,
  2. quantile $\widehat{v}^j$,
  3. controls $(\widehat{v}^1, \ldots, \widehat{v}^{j-1})$.
- 1 standard, 2 negligible, 3 nasty
- to avoid dealing with 3: non-analytic methods of inference
  - bootstrap
  - Bayesian bootstrap
  - subsampling

Back

Generalization of identification to $\dim(\alpha) > 1$

- $g(x,\alpha) = E[\partial_\alpha x | x, \alpha] \in \mathbb{R}^l$,
  $h(x,\alpha) = g(x,\alpha) \cdot f(x|\alpha)$
- $\nabla \cdot h := (\nabla \cdot h^1, \ldots, \nabla \cdot h^l)$
- Most results immediately generalize
- In particular

Theorem

$$\partial_\alpha f = -\nabla \cdot h \tag{17}$$

Theorem

The identified set for h is contained in

$$h^0 + \mathscr{H} \tag{18}$$

where

$$\mathscr{H} = \{\tilde{h} : \nabla \cdot \tilde{h} \equiv 0\}$$
$$h^{0j}(x,\alpha) = f(x|\alpha) \cdot \partial_\alpha Q(v^j | v^1, \ldots, v^{j-1}, \alpha)$$
$$v^j = F(x^j | x^1, \ldots, x^{j-1}, \alpha)$$

- open question: is this sharp?
- does the model restrict the set of admissible g?

A partial answer

Lemma

The system of PDEs

$$\partial_\alpha x(\alpha) = g(\alpha, x)$$
$$x(0) = x^0$$

has a local solution iff

$$\partial_\alpha g^j + \partial_x g^j \cdot g \tag{19}$$

is symmetric $\forall\, j$.
This solution is furthermore unique.

Proof: if: differentiation. only if: Frobenius' theorem. □

- cf. proof of sharpness in 1-d case
- **Q: what is the convex hull of all such** g**?**

Back