Learning by matching

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Introduction

- **Refugee resettlement** in the US: By resettlement agencies (like HIAS).
  - Small number of slots in various locations.
  - Refugees without ties: Distributed randomly.
- Ahani et al. (2021) / Annie MOORE:
  - Estimate refugee-location match effects on employment, *using past data*.
  - Find optimal matching, implement.
- This project: **Learning by matching.**
  An adaptive combinatorial allocation problem:
  - Refugees & locations get allocated to each other.
  - Feasibility constraints.
  - The returns of different matches are unknown.
  - The decision has to be made repeatedly.
- Similar to many other economic settings!

Sketch of setup

- There are J **matches**.
- Every period, our **action** is to choose (at most) M matches.
- Before the next period, we observe the **outcomes** of every chosen match.
- Our **reward** is the sum of the outcomes of the chosen matches.
- Our **objective** is to maximize the cumulative expected rewards.

Notation:

- Actions
$$a \in \mathcal{A} \subseteq \{a \in \{0,1\}^J : \|a\|_1 = M\}.$$
- Expected reward:
$$R(a) = \mathbf{E}[\langle a, Y_t \rangle | \Theta] = \langle a, \Theta \rangle.$$

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Thompson sampling

- Take a random action $a \in \mathcal{A}$, sampled according to the distribution

$$\mathbf{P}_t(A_t = a) = \mathbf{P}_t(A_t^* = a),$$

where $\mathbf{P}_t$ is the posterior at the beginning of period t.

- Introduced by Thompson (1933) for treatment assignment in adaptive experiments.

Regret bound

Theorem

Under the assumptions just stated,

$$\mathbf{E}_1\left[\sum_{t=1}^{T}\left(R(A^*) - R(A_t)\right)\right] \leq \sqrt{\frac{1}{2}JTM \cdot \left[\log\left(\frac{J}{M}\right) + 1\right]}.$$

Features of this bound:

- It holds in finite samples, there is no remainder.
- It does not depend on the prior distribution for Θ.
- It allows for prior distributions with arbitrary statistical dependence across the components of Θ.
- It implies that Thompson sampling achieves the efficient rate of convergence.

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Verbal description of this bound:

- The worst case expected regret (per unit) across all possible priors goes to 0 at a rate of 1 over the square root of the sample size, $T\cdot M$.
- The bound grows, as a function of the number of possible matches J, like $\sqrt{J}$ (ignoring the logarithmic term).
- Worst case regret per unit does not grow in the batch size M, despite the fact that action sets can be of size $\binom{J}{M}$!

Simulations for refugee matching

- Data for all refugees resettled by HIAS between January 2011 and December 2019.
- 8 demographic groups (types) based on
  - prime working age (25-54),
  - gender,
  - English-speaking.
- 17 affiliates (locations), with capacity constraints.
- Outcome Y_{jt}: Employed within 90 days of arrival.
- Simulations:
  - Calibrate success rates Θ_j for each type/affiliate combination.
  - Take actual capacity constraints.
  - Counterfactual matching using Thompson sampling.
  - Form posteriors using a hierarchical Bayesian model.

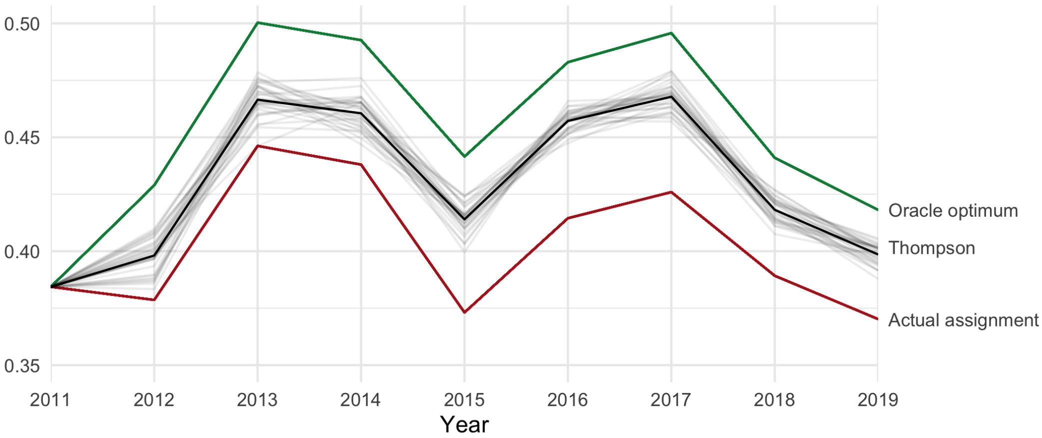
Hierarchical Bayesian model for match returns

$$Y_{jt} \sim \mathrm{Bernoulli}(\Theta_j),$$

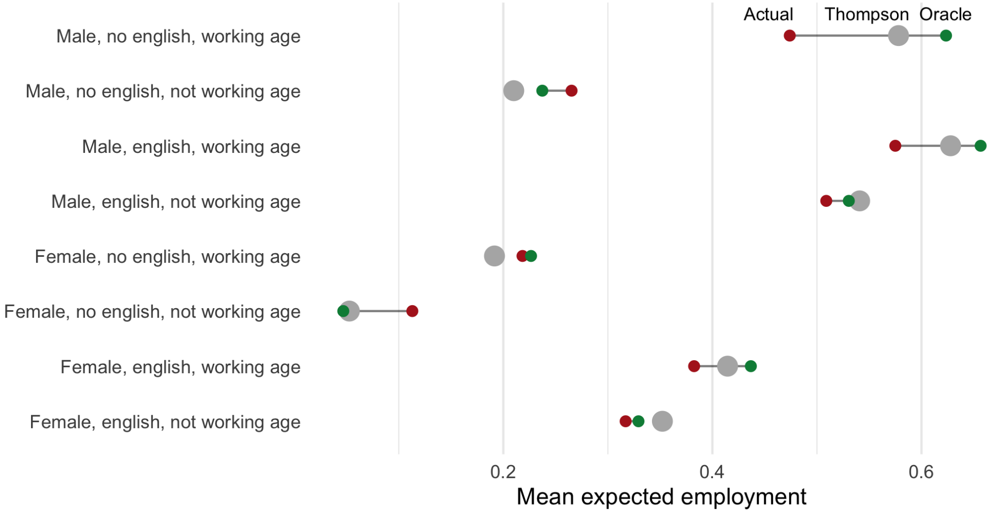
$$\Theta_j = \frac{1}{1 + \exp\left(-\left(\Gamma^{u}_{u_j} + \Gamma^{v}_{v_j} + \Gamma^{uv}_{u_j,v_j}\right)\right)},$$

$$\Gamma^{u}_{u_j} \sim N(0, \tau^2_{\Gamma^u}), \quad \Gamma^{v}_{v_j} \sim N(0, \tau^2_{\Gamma^v}), \quad \Gamma^{uv}_{u_j,v_j} \sim N(\mu, \tau^2_{\Gamma^{uv}}).$$

Simulated employment by year



Simulated employment by type



Thank you!