Adaptive treatment assignment in experiments for policy choice

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January 20, 2020

Introduction

The goal of many experiments is to inform policy choices:

1. **Job search assistance** for refugees:
   - Treatments: Information, incentives, counseling, ...
   - Goal: Find a policy that helps as many refugees as possible to find a job.
2. **Clinical trials**:
   - Treatments: Alternative drugs, surgery, ...
   - Goal: Find the treatment that maximizes the survival rate of patients.
3. Online **A/B testing**:
   - Treatments: Website layout, design, search filtering, ...
   - Goal: Find the design that maximizes purchases or clicks.
4. Testing **product design**:
   - Treatments: Various alternative designs of a product.
   - Goal: Find the best design in terms of user willingness to pay.

Example

- There are 3 treatments d.
- $d = 1$ is best, $d = 2$ is a close second, $d = 3$ is clearly worse. (But we don't know that beforehand.)
- You can potentially run the experiment in 2 waves.
- You have a fixed number of participants.
- After the experiment, you pick the best performing treatment for large scale implementation.

How should you design this experiment?

1. Conventional approach.
2. Bandit approach.
3. Our approach.

Conventional approach

Split the sample equally between the 3 treatments,
to get precise estimates for each treatment.

- After the experiment, it might still be hard to distinguish whether treatment 1 is best, or treatment 2.
- You might wish you had not wasted a third of your observations on treatment 3, which is clearly worse.

The conventional approach is

1. good if your goal is to get a precise estimate for each treatment.
2. not optimal if your goal is to figure out the best treatment.

Bandit approach

Run the experiment in **2 waves**
split the first wave equally between the 3 treatments.
Assign **everyone** in the second (last) wave to
the **best performing treatment** from the first wave.

- After the experiment, you have a lot of information on the d that performed best in wave 1, probably $d = 1$ or $d = 2$,
- but much less on the other one of these two.
- It would be better if you had split observations equally between 1 and 2.

The bandit approach is

1. good if your goal is to maximize the outcomes of participants.
2. not optimal if your goal is to pick the best policy.

Our approach

Run the experiment in **2 waves**
split the first wave equally between the 3 treatments.
Split the second wave between
the **two best performing** treatments from the first wave.

- After the experiment you have the maximum amount of information to pick the best policy.

Our approach is

1. good if your goal is to pick the best policy,
2. not optimal if your goal is to estimate the effect of all treatments, or to maximize the outcomes of participants.

Let θ^d denote the average outcome
that would prevail if everybody was assigned to treatment d.

What is the objective of your experiment?

1. Getting precise treatment effect estimators, powerful tests:

$$\text{minimize} \sum_d (\hat{\theta}^d - \theta^d)^2$$

$\Rightarrow$ Standard experimental design recommendations.

2. Maximizing the outcomes of experimental participants:

$$\text{maximize} \sum_i \theta^{D_i}$$

$\Rightarrow$ Multi-armed bandit problems.

3. Picking a welfare maximizing policy after the experiment:

$$\text{maximize } \theta^{d^*},$$

where d^* is chosen after the experiment.
$\Rightarrow$ This talk.

Preview of findings

- Adaptive designs improve expected welfare.
- Features of the optimal treatment assignment:
  - Shift toward better performing treatments over time.
  - But don't shift as much as for Bandit problems:
    We have no "exploitation" motive!
  - Asymptotically: Equalize power for comparisons
    of each suboptimal treatment to the optimal one.
- Fully optimal assignment is computationally challenging in large samples.
- We propose a simple **exploration sampling** algorithm.
  - Prove theoretically that it is rate-optimal for our problem,
    because it equalizes power across suboptimal treatments.
  - Show that it dominates alternatives in calibrated simulations.

Literature

- Adaptive designs in clinical trials:
    - Berry (2006), FDA (2018).
- Bandit problems:
    - Gittins index (optimal solution to some bandit problems): Weber (1992).
    - Regret bounds for bandit problems: Bubeck and Cesa-Bianchi (2012).
    - Thompson sampling: Russo et al. (2018).
- Best arm identification:
    - Rate-optimal (oracle) assignments: Glynn and Juneja (2004).
    - Poor rates of bandit algorithms: Bubeck et al. (2011),
    - Bayesian algorithms: Russo (2016).

  Key references for our theory results.
- Empirical examples for our simulations:
    - Ashraf et al. (2010),
    - Bryan et al. (2014),
    - Cohen et al. (2015).

Setup

Thompson sampling and exploration sampling

The rate optimal assignment

Exploration sampling is rate optimal

Calibrated simulations

Implementation in the field

Covariates and targeting

Setup

- Waves $t = 1, \ldots, T$, sample sizes N_t.
- Treatment $D \in \{1, \ldots, k\}$, outcomes $Y \in \{0, 1\}$.
- Potential outcomes Y^d.
- Repeated cross-sections:
  $(Y_{it}^0, \ldots, Y_{it}^k)$ are i.i.d. across both i and t.
- Average potential outcome:
  $$\theta^d = E[Y_{it}^d].$$
- Key choice variable:
  Number of units n_t^d assigned to $D = d$ in wave t.
- Outcomes:
  Number of units s_t^d having a "success" (outcome $Y = 1$).

Treatment assignment, outcomes, state space

- Treatment assignment in wave t: $\boldsymbol{n}_t = (n_t^1, \ldots, n_t^k)$.
- Outcomes of wave t: $\boldsymbol{s}_t = (s_t^1, \ldots, s_t^k)$.
- Cumulative versions:

$$M_t = \sum_{t' \leq t} N_{t'}, \qquad \boldsymbol{m}_t = \sum_{t' \leq t} \boldsymbol{n}_t, \qquad \boldsymbol{r}_t = \sum_{t' \leq t} \boldsymbol{s}_t.$$

- Relevant information for the experimenter in period $t+1$ is summarized by $\boldsymbol{m}_t$ and $\boldsymbol{r}_t$.
- Total trials for each treatment, total successes.

Design objective and Bayesian prior

- **Policy objective** $\theta^{d_T^*}$.
  - where d_T^* is chosen after the experiment.
- **Prior**
  - $\theta^d \sim Beta(\alpha_0^d, \beta_0^d)$, independent across d.
  - Posterior after period t: $\theta^d | \boldsymbol{m}_t, \boldsymbol{r}_t \sim Beta(\alpha_t^d, \beta_t^d)$

$$\alpha_t^d = \alpha_0^d + r_t^d$$
$$\beta_t^d = \beta_0^d + m_t^d - r_t^d.$$

- **Posterior expected social welfare**

  as a function of d:

$$\begin{aligned} SW_T(d) &= E[\theta^d | \boldsymbol{m}_T, \boldsymbol{r}_T], \\ &= \frac{\alpha_T^d}{\alpha_T^d + \beta_T^d}, \\ d_T^* &\in \underset{d}{\operatorname{argmax}}\ SW_T(d). \end{aligned}$$

Regret

- True optimal treatment: $d^{(1)} \in \arg\max_{d'} \theta^{d'}$.
- **Policy regret** when choosing treatment d:

$$\Delta^d = \theta^{d^{(1)}} - \theta^d.$$

- Maximizing expected social welfare is equivalent to minimizing the expected policy regret at T,

$$E[\Delta^d | \boldsymbol{m}_T, \boldsymbol{r}_T] = \theta^{d^{(1)}} - SW_T(d)$$

- **In-sample regret**: Objective considered in the bandit literature,

$$\tfrac{1}{M} \sum_{i,t} \Delta^{D_{it}}.$$

Different from policy regret $\Delta^{d_T^*}$!

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Thompson sampling

- **Thompson sampling**
  - Old proposal by Thompson (1933).
  - Popular in online experimentation.
- Assign each treatment with probability equal to the posterior probability that it is optimal.

$$p_t^d = P\left(d = \underset{d'}{\operatorname{argmax}}\ \theta^{d'} | \boldsymbol{m}_{t-1}, \boldsymbol{r}_{t-1}\right).$$

- Easily implemented: Sample draws $\widehat{\boldsymbol{\theta}}_{it}$ from the posterior, assign

$$D_{it} = \underset{d}{\operatorname{argmax}}\ \hat{\theta}_{it}^d.$$

- **Expected Thompson sampling**
  - Straightforward modification for the batched setting.
  - Assign non-random shares p_t^d of each wave to treatment d.

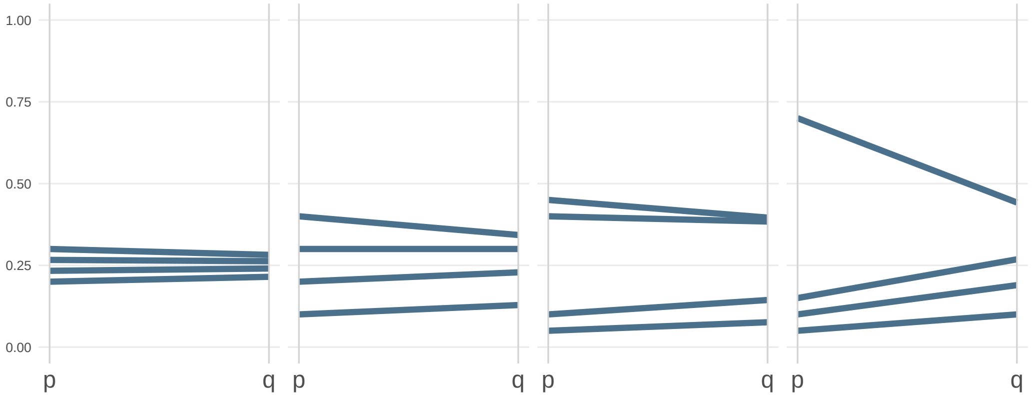
Exploration sampling

- Agrawal and Goyal (2012) proved that Thompson-sampling is rate-optimal for the multi-armed bandit problem.
- It is not for our policy choice problem!
- We propose the following modification.
- **Exploration sampling**:
  Assign shares q_t^d of each wave to treatment d, where

$$q_t^d = S_t \cdot p_t^d \cdot (1 - p_t^d),$$
$$S_t = \frac{1}{\sum_d p_t^d \cdot (1 - p_t^d)}.$$

- This modification
  1. yields rate-optimality (theorem coming up), and
  2. improves performance in our simulations.

Illustration of the mapping from Thompson to exploration sampling



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Implementation in the field

Covariates and targeting

The rate-optimal assignment: Lemma 1

Denote the estimated success rate of d at time T by $\hat{\theta}_T^d = \frac{1+r_T^d}{2+m_T^d}$.

The rate of convergence to zero of expected policy regret

$$\mathrm{R(T)} = \sum_d \Delta^d \cdot P\left(\underset{d'}{\operatorname{argmax}}\ \hat{\theta}_T^{d'} = d\right)$$

is equal to the slowest rate of convergence Γ^d across $d \neq d^{(1)}$ for the probability of d being estimated to be better than $d^{(1)}$.

Lemma

- *Assume that the optimal policy $d^{(1)}$ is unique. Suppose that for all d*

$$\lim_{T\to\infty} -\frac{1}{NT} \log P\left(\hat{\theta}_T^d > \hat{\theta}_T^{d^{(1)}}\right) = \Gamma^d.$$

- *Then*

$$\lim_{T\to\infty}\left(-\frac{1}{NT}\log \mathrm{R(T)}\right) = \min_{d\neq d^{(1)}} \Gamma^d.$$

The rate-optimal assignment: Lemma 2

From Glynn and Juneja (2004):

- Characterize Γ^d as a function of the treatment allocation share for each d, $\bar{q}^d$.
- The posterior probability p_T^d of d being optimal converges at the same rate Γ^d.

Lemma

Suppose that $\bar{q}_T^d = m_T^d/(NT)$ converges to $\bar{q}^d$ for all d, with $\bar{q}^{d^{(1)}} = 1/2$. Then

1. $\lim_{T\to\infty} -\frac{1}{NT} \log P\left(\hat{\theta}_T^d > \hat{\theta}_T^{d^{(1)}}\right) = \Gamma^d$, *and*
2. $\operatorname{plim}_{T\to\infty} -\frac{1}{NT} \log p_T^d = \Gamma^d$,

where

$$\Gamma^d = G^d(\bar{q}^d)$$

for a function $G^d : [0,1] \to \mathbb{R}$
that is finitely valued, continuous, strictly increasing in $\bar{q}^d$, and satisfies $G^d(0) = 0$.

The rate-optimal assignment: Lemma 3

- Characterize the allocation of observations across the treatments d which maximizes the rate of R(T).
- Our main result shows that exploration sampling converges to this allocation.

Lemma

The rate-optimal allocation $\bar{\boldsymbol{q}}$, subject to the constraint $\bar{q}^{d^{(1)}} = 1/2$, is given by the unique solution to the system of equations

$$\sum_{d \neq d^{(1)}} \bar{q}^d = 1/2 \quad \mathit{and} \quad G^d(\bar{q}^d) = \Gamma^* > 0 \text{ for all } d \neq d^{(1)} \tag{1}$$

for some Γ^*. No other allocation, subject to the constraint $\bar{q}^{d^{(1)}} = 1/2$, can achieve a faster rate of convergence of R(T) *than Γ^*.*

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The rate optimal assignment

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Calibrated simulations

Implementation in the field

Covariates and targeting

Thompson sampling – results from the literature

- **In-sample regret** (bandit objective):
  $\sum_{t=1}^{T} \Delta^d$, where $\Delta^d = \max_{d'} \theta^{d'} - \theta^d$.
- Agrawal and Goyal (2012) (Theorem 2): For Thompson sampling,

$$\lim_{T \to \infty} E\left[\frac{\sum_{t=1}^{T} \Delta^d}{\log T}\right] \leq \left(\sum_{d \neq d^*} \frac{1}{(\Delta^d)^2}\right)^2.$$

- Lai and Robbins (1985):
  No adaptive experimental design can do better than this $\log T$ rate.
- Thompson sampling only assigns a share of units of order $\log(M)/M$ to treatments other than the optimal treatment.

Results from the literature continued

- This is good for in-sample welfare, bad for learning:
  We stop learning about suboptimal treatments very quickly.
- Bubeck et al. (2011) Theorem 1 implies:
  Any algorithm that achieves $\log(M)/M$ rate for in-sample regret
  (such as Thompson sampling)
  can at most achieve **polynomial convergence** for policy regret!
- By contrast (easy to show): Any algorithm that assigns shares
  converging to non-zero shares for each treatment
  achieves **exponential convergence** for our objective.
- Our result (next slide): Exploration sampling achieves the
  (constrained) best exponential rate.

Exploration sampling is rate optimal

Theorem

Consider exploration sampling in a setting with fixed wave size $N_t = N \geq 1$. Assume that $\theta^{d^{(1)}} < 1$ and that the optimal policy $d^{(1)}$ is unique. As $T \to \infty$, the following holds:

1. *The share of observations $\bar{q}_T^{d^{(1)}}$ assigned to the best treatment converges in probability to $1/2$.*
2. *The share of observations $\bar{q}_T^d$ assigned to treatment d converges in probability to a non-random share $\bar{q}^d$ for all $d \neq d^{(1)}$.*
   $\bar{q}^d$ is such that $-\frac{1}{NT}\log p_t^d \to^p \Gamma^*$ for some $\Gamma^* > 0$ that is constant across $d \neq d^{(1)}$.
3. *Expected policy regret converges to 0 at the same rate Γ^*, that is, $-\frac{1}{NT}\log \mathrm{R(T)} \to^p \Gamma^*$.*
   No other assignment shares $\bar{q}^d$ exist for which $\bar{q}^{d^{(1)}} = 1/2$ and $\mathrm{R(T)}$ goes to 0 at a faster rate than Γ^*.

Sketch of proof

Our proof draws on several Lemmas of Glynn and Juneja (2004) and Russo (2016).
Proof steps:

1. Each treatment is assigned infinitely often.
   $\Rightarrow p_T^d$ goes to 1 for the optimal treatment and to 0 for all other treatments.
2. Claim 1 then follows from the definition of exploration sampling.
3. Claim 2: Suppose p_t^d goes to 0 at a faster rate for some d.
   Then exploration sampling stops assigning this d.
   This allows the other treatments to "catch up."
4. Claim 3: Balancing the rate of convergence implies efficiency.
   This follows from the Lemmas discussed before.

Calibrated simulations

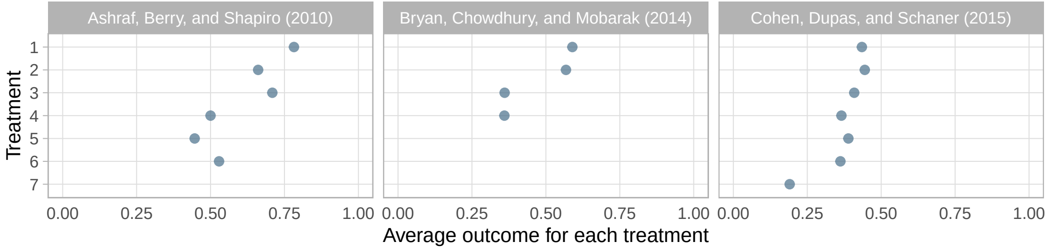
- Simulate data calibrated to estimates of 3 published experiments.
- Set $\boldsymbol{\theta}$ equal to observed average outcomes for each stratum and treatment.
- Total sample size same as original.

Ashraf, N., Berry, J., and Shapiro, J. M. (2010). Can higher prices stimulate product use? Evidence from a field experiment in Zambia.
American Economic Review, 100(5):2383–2413

Bryan, G., Chowdhury, S., and Mobarak, A. M. (2014). Underinvestment in a profitable technology: The case of seasonal migration in Bangladesh.
Econometrica, 82(5):1671–1748

Cohen, J., Dupas, P., and Schaner, S. (2015). Price subsidies, diagnostic tests, and targeting of malaria treatment: evidence from a randomized controlled trial.
American Economic Review, 105(2):609–45

Calibrated parameter values



Treatment arms labeled 1 up to 7:

- Ashraf et al. (2010): Kw 300 - 800 price for water disinfectant.
- Bryan et al. (2014): Migration incentives - cash, credit, information, and control.
- Cohen et al. (2015): Price of Ksh 40, 60, and 100 for malaria tablets, each with and without free malaria test, and control of Ksh 500.

Summary of simulation findings

- With two waves, relative to non-adaptive assignment:
  - Thompson reduces average policy regret by 15-58 %,
  - exploration sampling by 21-67 %.
- Similar pattern for the probability of choosing the optimal treatment.
- Gains increase with the number of waves, given total sample size.
  - Up to 85% for exploration sampling with 10 waves for Ashraf et al. (2010).
- Gains largest for Ashraf et al. (2010),
  followed by Cohen et al. (2015),
  and smallest for Bryan et al. (2014).
- For in-sample regret, Thompson is best,
  followed closely by exploration sampling.

Ashraf, Berry, and Shapiro (2010)

Statistic	2 waves	4 waves	10 waves
Average policy regret			
exploration sampling	0.0017	0.0010	0.0008
expected Thompson	0.0022	0.0014	0.0013
non-adaptive	0.0051	0.0050	0.0051
Share optimal			
exploration sampling	0.978	0.987	0.989
expected Thompson	0.971	0.981	0.982
non-adaptive	0.933	0.935	0.933
Average in-sample regret			
exploration sampling	0.1126	0.0828	0.0701
expected Thompson	0.1007	0.0617	0.0416
non-adaptive	0.1776	0.1776	0.1776
Units per wave	502	251	100

Bryan, Chowdhury, and Mobarak (2014)

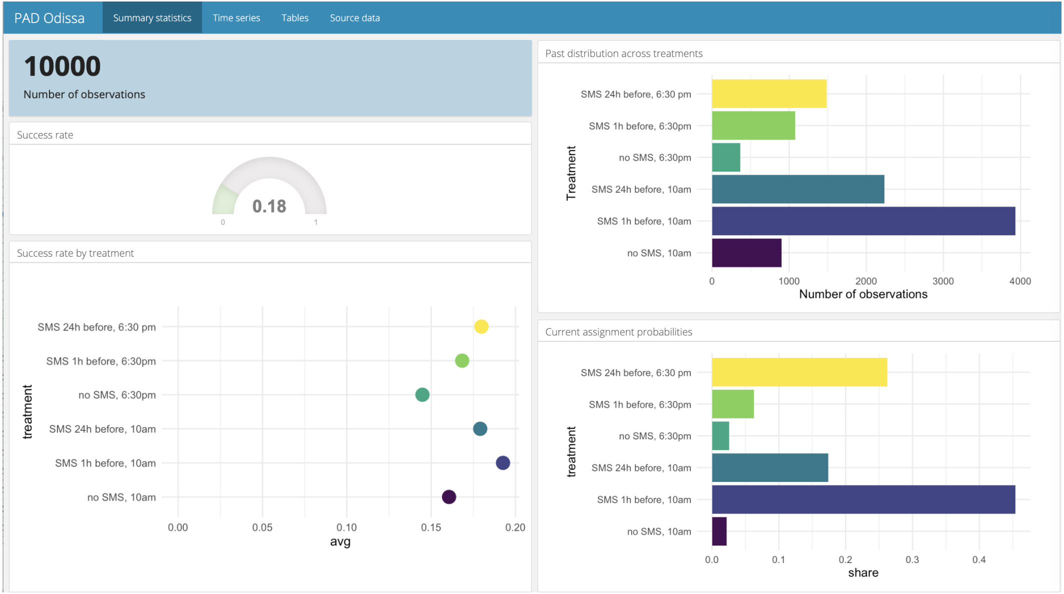
Statistic	2 waves	4 waves	10 waves
Average policy regret			
exploration sampling	0.0045	0.0041	0.0039
expected Thompson	0.0048	0.0044	0.0043
non-adaptive	0.0055	0.0054	0.0054
Share optimal			
exploration sampling	0.792	0.812	0.820
expected Thompson	0.777	0.795	0.801
non-adaptive	0.747	0.748	0.749
Average in-sample regret			
exploration sampling	0.0655	0.0386	0.0254
expected Thompson	0.0641	0.0359	0.0205
non-adaptive	0.1201	0.1201	0.1201
Units per wave	935	467	187

Cohen, Dupas, and Schaner (2015)

Statistic	2 waves	4 waves	10 waves
Average policy regret			
exploration sampling	0.0070	0.0063	0.0060
expected Thompson	0.0074	0.0065	0.0061
non-adaptive	0.0086	0.0087	0.0085
Share optimal			
exploration sampling	0.567	0.586	0.592
expected Thompson	0.560	0.582	0.589
non-adaptive	0.526	0.524	0.529
Average in-sample regret			
exploration sampling	0.0489	0.0374	0.0314
expected Thompson	0.0467	0.0345	0.0278
non-adaptive	0.0737	0.0737	0.0737
Units per wave	1080	540	216

Implementation in the field

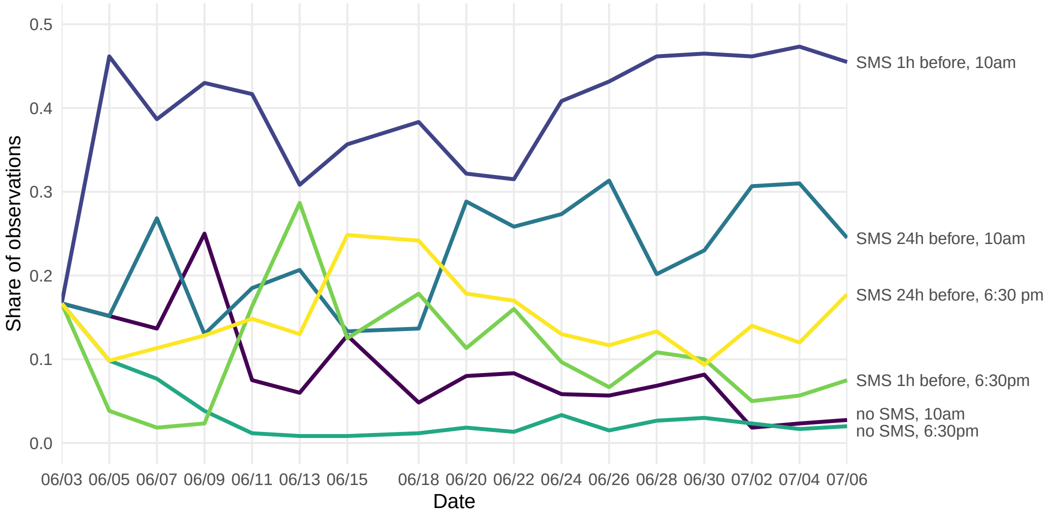
- NGO Precision Agriculture for Development (PAD), and Government of Odisha, India.
- Enrolling rice farmers into customized advice service by mobile phone.
- Waves of 600 farmers called through automated service; total of 10K calls.
- Outcome: did the respondent answer the enrollment questions?


PAD Odissa
Summary statistics
Time series
Tables
Source data
10000
Number of observations
Success rate
0.18
0
1
Success rate by treatment
SMS 24h before, 6:30 pm
SMS 1h before, 6:30pm
no SMS, 6:30pm
SMS 24h before, 10am
SMS 1h before, 10am
no SMS, 10am
treatment
0.00
0.05
0.10
0.15
0.20
avg
Past distribution across treatments
SMS 24h before, 6:30 pm
SMS 1h before, 6:30pm
no SMS, 6:30pm
SMS 24h before, 10am
SMS 1h before, 10am
no SMS, 10am
Treatment
0
1000
2000
3000
4000
Number of observations
Current assignment probabilities
SMS 24h before, 6:30 pm
SMS 1h before, 6:30pm
no SMS, 6:30pm
SMS 24h before, 10am
SMS 1h before, 10am
no SMS, 10am
treatment
0.0
0.1
0.2
0.3
0.4
share

Outcomes and posterior parameters

Treatment		Outcomes			Posterior		
Call time	SMS alert	m_T^d	r_T^d	r_T^d / m_T^d	mean	SD	p_T^d
10am	-	903	145	0.161	0.161	0.012	0.009
10am	1h ahead	3931	757	0.193	0.193	0.006	0.754
10am	24h ahead	2234	400	0.179	0.179	0.008	0.073
6:30pm	-	366	53	0.145	0.147	0.018	0.011
6:30pm	1h ahead	1081	182	0.168	0.169	0.011	0.027
6:30 pm	24h ahead	1485	267	0.180	0.180	0.010	0.126

Assignment shares over time



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Extension: Covariates and treatment targeting

- Suppose now that
  1. We additionally observe a (discrete) covariate X.
  2. The policy to be chosen can **target treatment** by X.
- How to adapt exploration sampling to this setting?
- Solution: Hierarchical Bayes model,
  to optimally combine information across strata.
- Example of a **hierarchical Bayes** model:

$$
\begin{aligned}
Y^d | X = x, \theta^{dx}, (\alpha_0^d, \beta_0^d) &\sim Ber(\theta^{dx}) \\
\theta^{dx} | (\alpha_0^d, \beta_0^d) &\sim Beta(\alpha_0^d, \beta_0^d) \\
(\alpha_0^d, \beta_0^d) &\sim \pi,
\end{aligned}
$$

- No closed form posterior, but can use Markov Chain Monte Carlo to sample from posterior.

MCMC sampling from the posterior

Combining Gibbs sampling & Metropolis-Hasting

- Iterate across replication draws ρ:
  1. **Gibbs** step: Given $\boldsymbol{\alpha}_{\rho-1}$ and $\boldsymbol{\beta}_{\rho-1}$,
     - draw $\theta^{dx} \sim Beta(\alpha^d_{\rho-1} + s^{dx}, \beta^d_{\rho-1} + m^{dx} - s^{dx})$.
  2. **Metropolis** step: Given $\boldsymbol{\beta}_{\rho-1}$ and $\boldsymbol{\theta}_\rho$,
     - draw $\alpha^d_\rho \sim$ (symmetric proposal distribution).
     - Accept if an independent uniform is less than the ratio of the posterior for the new draw, relative to the posterior for $\alpha^d_{\rho-1}$.
     - Otherwise set $\alpha^d_\rho = \alpha^d_{\rho-1}$.
  3. **Metropolis** step: Given $\boldsymbol{\theta}_\rho$ and $\boldsymbol{\alpha}_\rho$,
     - proceed as in 2, for β^d_ρ.
- This converges to a stationary distribution such that

$$P\left(d = \underset{d'}{\operatorname{argmax}}\ \theta^{d'x} | \boldsymbol{m}_t, \boldsymbol{r}_t\right) = \underset{R\to\infty}{\operatorname{plim}} \frac{1}{R} \sum_{\rho=1}^{R} \mathbf{1}\left(d = \underset{d'}{\operatorname{argmax}}\ \theta^{d'x}_\rho\right).$$

Conclusion

- Different objectives lead to different optimal designs:
  1. Treatment effect estimation / testing: Conventional designs.
  2. In-sample regret: Bandit algorithms.
  3. Post-experimental policy choice: This talk.
- If the experiment can be implemented in multiple waves, adaptive designs for policy choice
  1. significantly increase welfare,
  2. by focusing attention in later waves on the best performing policy options,
  3. but not as much as bandit algorithms.
  4. Asymptotically: Equalize power for comparisons of each suboptimal treatment to the optimal one.
- Implementation of our proposed procedure is easy and fast, and easily adapted to new settings:
  - Hierarchical priors,
  - non-binary outcomes...
- Interactive dashboard for treatment assignment: `https://maxkasy.shinyapps.io/exploration_sampling_dashboard/`

Thank you!