

How to run an adaptive field experiment

Maximilian Kasy

September 2020

Is experimentation on humans ethical?

Deaton (2020):

Some of the RCTs done by western economists on extremely poor people [...] could not have been done on American subjects.

It is particularly worrying if the research addresses questions in economics that appear to have no potential benefit for the subjects.

Do our experiments have enough power?

Ioannidis et al. (2017):

We survey 159 empirical economics literatures that draw upon 64,076 estimates of economic parameters reported in more than 6,700 empirical studies. Half of the research areas have nearly 90% of their results under-powered. The median statistical power is 18%, or less.

Are experimental sites systematically selected?

Andrews and Oster (2017):

[...] the selection of locations is often non-random in ways that may influence the results. [...] this concern is particularly acute when we think researchers select units based in part on their predictions for the treatment effect.

Claim: Adaptive experimental designs can partially address these concerns

1. Ethics and **participant welfare**:

Bandit algorithms are designed to maximize participant outcomes, by shifting to the best performing options at the right speed.

2. **Statistical power** and publication bias:

Exploration Sampling, introduced in Kasy and Sautmann (2020), is designed to maximize power for distinguishing the best policy, by focusing attention on competitors for the best option.

3. **Political economy**, site selection, and external validity:

Related to the ethical concerns: Design experiments that maximize the stakeholders' goals (where appropriate). This might allow us to reduce site selectivity, by making experiments more widely acceptable.

What is adaptivity?

- Suppose your experiment takes place over time.
- Not all units are assigned to treatments at the same time.
- You can observe outcomes for some units before deciding on the treatment for later units.
- Then treatment assignment can depend on earlier outcomes, and thus be *adaptive*.

Why adaptivity?

- Using more information is always better than using less information, when making (treatment assignment) decisions.
- Suppose you want to
 1. **Help participants**
⇒ Shift toward the best performing option.
 2. **Learn the best treatment**
⇒ Shift toward best candidate options, to maximize power.
 3. **Estimate treatment effects**
⇒ Shift toward treatment arms with higher variance.
- Adaptivity allows us to achieve better performance with smaller sample sizes.

When is adaptivity useful?

1. Time till outcomes are realized:

- Seconds? (Clicks on a website.) Decades? (Alzheimer prevention.) Intermediate? (Many settings in economics.)
- Even when outcomes take months, adaptivity can be quite feasible.
- Splitting the sample into a small number of waves already helps a lot.
- Surrogate outcomes (discussed later) can shorten the wait time.

2. Sample size and effect sizes:

- Algorithms can adapt, if they can already learn something before the end of the experiment.
- In very underpowered settings, the benefits of adaptivity are smaller.

3. Technical feasibility:

- Need to create a pipeline:
Outcome measurement - belief updating - treatment assignment.
- With apps and mobile devices for fieldworkers, that is quite feasible, but requires some engineering.

Papers this talk is based on

- Kasy, M. and Sautmann, A. (2020).
Adaptive treatment assignment in experiments for policy choice.
Forthcoming, Econometrica
- Caria, S., Gordon, G., Kasy, M., Osman, S., Quinn, S., and Teytelboym, A. (2020).
**An Adaptive Targeted Field Experiment:
Job Search Assistance for Refugees in Jordan.**
Working paper.
- Kasy, M. and Teytelboym, A. (2020a).
Adaptive combinatorial allocation.
Work in progress.
- Kasy, M. and Teytelboym, A. (2020b).
Adaptive targeted disease testing.
Forthcoming, Oxford Review of Economic Policy.

Literature

- Statistical decision theory:
Berger (1985),
Robert (2007).
- Non-parametric Bayesian methods:
Ghosh and Ramamoorthi (2003),
Williams and Rasmussen (2006),
Ghosal and Van der Vaart (2017).
- Stratification and re-randomization:
Morgan and Rubin (2012),
Athey and Imbens (2017).
- Adaptive designs in clinical trials:
Berry (2006),
FDA et al. (2018).
- Bandit problems:
Weber et al. (1992),
Bubeck and Cesa-Bianchi (2012),
Russo et al. (2018).
- Regret bounds:
Agrawal and Goyal (2012),
Russo and Van Roy (2016).
- Best arm identification:
Glynn and Juneja (2004),
Bubeck et al. (2011),
Russo (2016).
- Bayesian optimization:
Powell and Ryzhov (2012),
Frazier (2018).
- Reinforcement learning:
Ghavamzadeh et al. (2015),
Sutton and Barto (2018).
- Optimal taxation:
Mirrlees (1971),
Saez (2001),
Chetty (2009),
Saez and Stantcheva (2016).

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Setup

- Waves $t = 1, \dots, T$, sample sizes N_t .
- Treatment $D \in \{1, \dots, k\}$, outcomes $Y \in [0, 1]$, covariate $X \in \{1, \dots, n_x\}$.
- Potential outcomes Y^d .
- Repeated cross-sections:
($Y_{it}^1, \dots, Y_{it}^k, X_{it}$) are i.i.d. across both i and t .
- Average potential outcomes:

$$\theta^{dx} = E[Y_{it}^d | X_{it} = x].$$

Adaptive targeted assignment

- The algorithms I will discuss are Bayesian.
- Given all the information available at the beginning of wave t , form posterior beliefs P_t over θ .
- Based on these beliefs, decide what share p_t^{dx} of stratum x will be assigned to treatment d in wave t .
- How you should to pick these assignment shares depends on the objective you try to maximize.

Bayesian updating

- In **simple** cases, posteriors are easy to calculate in closed form.
 - Example: Binary outcomes, no covariates.
 - Assume that $Y \in \{0, 1\}$, $Y_t^d \sim \text{Ber}(\theta^d)$. Start with a uniform prior for θ on $[0, 1]^k$.
 - Then the posterior for θ^d at time $t + 1$ is a *Beta* distribution with parameters

$$\alpha_t^d = 1 + T_t^d \cdot \bar{Y}_t^d, \quad \beta_t^d = 1 + T_t^d \cdot (1 - \bar{Y}_t^d).$$

- In more complicated cases, simulate from the posterior using MCMC (more later).
 - For well chosen **hierarchical** priors:
 - θ^{dx} is estimated as a **weighted average** of the observed success rate for d in x and the observed success rates for d across all other strata.
 - The **weights** are determined **optimally** by the observed amount of heterogeneity across all strata as well as the available sample size in a given stratum.

Objective I: Participant welfare

- **Regret:** Difference in average outcomes from decision d versus the optimal decision,

$$\Delta^{dx} = \max_{d'} \theta^{d'x} - \theta^{dx}.$$

- Average **in-sample regret**:

$$\bar{R}_\theta(T) = \frac{1}{\sum_t N_t} \sum_{i,t} \Delta^{D_{it}X_{it}}.$$

- **Thompson sampling**
 - Old proposal by Thompson (1933).
 - Popular in online experimentation.
- **Assign** each treatment with **probability** equal to the posterior probability that it is optimal, given $X = x$ and given the information available at time t .

$$p_t^{dx} = P_t \left(d = \operatorname{argmax}_{d'} \theta^{d'x} \right).$$

Thompson sampling is efficient for participant welfare

- **Lower bound** (Lai and Robbins, 1985):

Consider the Bandit problem with binary outcomes and any algorithm. Then

$$\liminf_{T \rightarrow \infty} \frac{T}{\log(T)} \bar{R}_\theta(T) \geq \sum_d \frac{\Delta^d}{kl(\theta^d, \theta^*)},$$

where $kl(p, q) = p \cdot \log(p/q) + (1 - p) \cdot \log((1 - p)/(1 - q))$.

- **Upper bound for Thompson sampling** (Agrawal and Goyal, 2012):
Thompson sampling achieves this bound, i.e.,

$$\liminf_{T \rightarrow \infty} \frac{T}{\log(T)} \bar{R}_\theta(T) = \sum_d \frac{\Delta^d}{kl(\theta^d, \theta^*)}.$$

Mixed objective: Participant welfare and point estimates

- Suppose you care about both participant welfare, and precise point estimates / high power for all treatments.
- In Caria et al. (2020), we introduce **Tempered Thompson sampling**: Assign each treatment with probability equal to

$$\tilde{p}_t^{dx} = (1 - \gamma) \cdot p_t^{dx} + \gamma/k.$$

Compromise between full randomization and Thompson sampling.

Tempered Thompson trades off participant welfare and precision

We show in Caria et al. (2020):

- **In-sample regret** is (approximately) proportional to the share γ of observations fully randomized.
- The **variance** of average potential outcome estimators is proportional
 - to $\frac{1}{\gamma/k}$ for sub-optimal d ,
 - to $\frac{1}{(1-\gamma)+\gamma/k}$ for conditionally optimal d .
- The variance of **treatment effect** estimators, comparing the conditional optimum to alternatives, is therefore decreasing in γ .
- An **optimal** choice of γ **trades off** regret and estimator variance.

Objective II: Policy choice

- Suppose you will **choose a policy** after the experiment, based on posterior beliefs,

$$d_T^* \in \operatorname{argmax}_d \hat{\theta}_T^d, \quad \hat{\theta}_T^d = E_T[\theta^d].$$

- Evaluate experimental designs based on expected welfare (ex ante, given θ).
- Equivalently, **expected policy regret**

$$R_\theta(T) = \sum_d \Delta^d \cdot P(d_T^* = d), \quad \Delta^d = \max_{d'} \theta^{d'} - \theta^d.$$

- In Kasy and Sautmann (2020), we introduce **Exploration sampling**:
Assign shares q_t^d of each wave to treatment d , where

$$q_t^d = S_t \cdot p_t^d \cdot (1 - p_t^d),$$
$$p_t^d = P_t \left(d = \operatorname{argmax}_{d'} \theta^{d'} \right), \quad S_t = \frac{1}{\sum_d p_t^d \cdot (1 - p_t^d)}.$$

Exploration sampling is efficient for policy choice

- We show in Kasy and Sautmann (2020) (under mild conditions):
 - The posterior probability p_t^d that each treatment is optimal goes to 0 at the same rate for all sub-optimal treatments.
 - **Policy regret** also goes to 0 at the same rate.
 - No other algorithm can achieve a faster rate.
- Key intuition of proof: Equalizing power.
 1. Suppose p_t^d goes to 0 at a faster rate for some d .
Then exploration sampling stops assigning this d .
This allows the other treatments to “catch up.”
 2. Balancing the rate of convergence implies efficiency.

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Inference

- Inference has to take into account adaptivity, in general.
- Example:
 - Flip a fair coin.
 - If head, flip again, else stop.
 - Probability distribution: 50% tail-stop, 25% head-tail, 25% head-head.
 - Expected share of heads?

$$.5 \cdot 0 + .25 \cdot .5 + .25 \cdot 1 = .375 \neq .5.$$

- But:
 1. **Bayesian** inference works without modification.
 2. **Randomization tests** can be modified to work in adaptive settings.
 3. **Standard inference** (e.g., t-tests) works under some conditions.

Bayesian inference

- The likelihood, and thus the posterior, are **not affected by adaptive** treatment assignment.
- Claim: The likelihood of $(D_1, \dots, D_M, Y_1, \dots, Y_M)$ equals $\prod_i P(Y_i|D_i, \theta)$, up to a constant that does not depend on θ .
- Proof: Denote $H_i = (D_1, \dots, D_{i-1}, Y_1, \dots, Y_{i-1})$. Then

$$\begin{aligned} P(D_1, \dots, D_M, Y_1, \dots, Y_M | \theta) &= \prod_i P(D_i, Y_i | H_i, \theta) \\ &= \prod_i P(D_i | H_i, \theta) \cdot P(Y_i | D_i, H_i, \theta) \\ &= \prod_i P(D_i | H_i) \cdot P(Y_i | D_i, \theta). \end{aligned}$$

Randomization inference

- Strong null hypothesis: $Y_i^1 = \dots = Y_i^k$.
- Under this null, it is easy to re-simulate the treatment assignment:
Just let your assignment algorithm run with the data, switching out the treatments.
- Do this many times, re-calculate the test statistic each time.
- Take the $1 - \alpha$ quantile across simulations as critical value.
- This delivers **finite-sample exact inference** for any adaptive assignment scheme.

T-tests and F-tests

- As shown above, sample averages in treatment arms are, in general, biased, cf. Hadad et al. (2019).
- But: **Under some conditions**, the **bias is negligible** in large samples.
- In particular, suppose
 1. $\left(\sum_{i,t} \mathbf{1}(D_{it} = d)\right) / \tilde{N}_T^d \rightarrow^p 1$
 2. \tilde{N}_T^d is non-random and goes to ∞ .
- Then the standard law of large numbers and central limit theorem apply. T-tests can ignore adaptivity (Melfi and Page, 2000).
- This works for Exploration Sampling, Tempered Thompson sampling: Assignment shares are bounded away from 0.
- This does not work for many Bandit algorithms (e.g. Thompson sampling): Assignment shares for sub-optimal treatments go to 0 too fast.

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Data pipeline

Typical **cycle** for one wave of the experiment:

1. On a **central machine**, update the prior based on available data.
2. Calculate treatment assignment probabilities for each stratum.
3. Upload these to some **web-server**.
4. Field workers encounter participants, and enter participant covariates on a **mobile device**.
5. The mobile device assigns a treatment to participants, based on their covariates and the downloaded assignment probabilities.
6. A bit later, outcome data are collected, and transmitted to the central machine.

This is not infeasible, but it requires careful planning.

All steps should be **automated** for smooth implementation!

Surrogate outcomes

- We don't always observe the desired outcomes / measures of welfare quickly enough – or at all.
- Potential solution: **Surrogate** outcomes (Athey et al., 2019):
 - Suppose we want to maximize Y , but only observe other outcomes W , which satisfy the **surrogacy condition**

$$D \perp (Y^1, \dots, Y^d) | W.$$

- This holds **if all causal pathways** from D to Y **go through** W .
- Let $\hat{y}(W) = E[Y|W]$, estimated from auxiliary data. Then

$$\begin{aligned} E[Y|D] &= E[E[Y|D, W]|D] \\ &= E[E[Y|W]|D] = E[\hat{y}(W)|D]. \end{aligned}$$

- Implication: We can design algorithms that target maximization of $\hat{y}(W)$, and they will achieve the same objective.

Choice of prior

- One option: Informative prior, based on prior data or expert beliefs.
- I recommend instead: Default priors that are
 1. **Symmetric**: Start with exchangeable treatments, strata.
 2. **Hierarchical**: Model heterogeneity of effects across treatments, strata.
Learn “hyper-parameters” (levels and degree of heterogeneity) from the data.
⇒ Bayes estimates will be based on optimal partial pooling.
 3. **Diffuse**: Make your prior for the hyper-parameters uninformative.
- Example:

$$\begin{aligned}Y_{it}^d | (X_{it} = x, \theta^{dx}, \alpha^d, \beta^d) &\sim \text{Ber}(\theta^{dx}), \\ \theta^{dx} | (\alpha^d, \beta^d) &\sim \text{Beta}(\alpha^d, \beta^d), \\ (\alpha^d, \beta^d) &\sim \pi.\end{aligned}$$

MCMC sampling from the posterior

- For hierarchical models, posterior probabilities such as p_t^{dx} can be calculated by sampling from the posterior using **Markov Chain Monte Carlo**.
- General purpose Bayesian packages such as **Stan** make this easy:
 - Just specify your likelihood and prior.
 - The package takes care of the rest, using “Hamiltonian Monte Carlo.”
- Alternatively, do it “**by hand**” (e.g. using our code):
 - Combine Gibbs sampling & Metropolis-Hasting.
 - Given the hyper-parameters, sample from closed-form posteriors for θ .
 - Given θ , sample hyper-parameters using Metropolis (accept/reject) steps.

The political economy of experimentation

- Experiments often involve some **conflict of interest**, that might prevent experimentation where it could be useful.
 - Academic experimenters:
“We want to get estimates that we can publish.”
 - Implementation partners:
“We know what’s best, so don’t prevent us from helping our clients.”
- Adaptive designs can **partially resolve** these conflicts
 1. Maintain controlled treatment assignment,
 2. but choose assignment probabilities to maximize stakeholder objectives.
- Conflicts can of course remain.
e.g. Which outcomes to maximize? Choose carefully!

Conclusion

- Using adaptive designs in field experiments can have great **benefits**:
 1. More ethical, by helping participants as much as possible.
 2. Better power for a given sample size, by targeting policy learning.
 3. More acceptable to stakeholders, by aligning design with their objectives.
- Adaptive designs are practically **feasible**:

We have implemented them in the field.
E.g., labor market interventions for Syrian refugees in Jordan,
and agricultural outreach for subsistence farmers in India.
- Implementation requires learning some new **tools**.
 - I have developed some software to facilitate implementation.
 - Interactive apps for treatment assignment, and source code for various designs.

`https://maxkasy.github.io/home/code-and-apps/`

Thank you!