

Nonparametric inference on the number of equilibria - Web Appendix

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1 Overview

This Web Appendix presents tables and figures supplementing those shown in “Nonparametric inference on the number of equilibria”.

Section 2 displays further Monte Carlo evidence. Figures 1 through 4 show kernel density plots of \hat{Z} for the sequences of experiments described in Appendix A.

Section 3 shows empirical results on neighborhood tipping for all MSAs in the sample. Tables 1 through 6 show confidence sets for $Z(g)$ for all MSAs in the dataset, where g is estimated by quantile regression of the change in minority share over a decade on the initial minority share for the quantiles .2, .5 and .8. Regression bandwidth τ is $n^{-.2}$, σ is chosen as .04. Confidence sets are based on t-statistics using bootstrapped bias and standard errors.

Tables 7 through 12 show similar confidence sets for $Z(g)$ for all MSAs in the dataset, where g is estimated by quantile regression of the change in the non-Hispanic, white population over a decade, divided by initial total population, on the initial minority share for the quantiles .2, .5 and .8. Regression bandwidth τ is $n^{-.2}$, σ is chosen as .05 times the maximal change.

Section 4, finally, presents an application of the inference procedure to data on economic growth.

2 Further Monte Carlo results

3 Empirical results for all MSAs in the sample

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Figure 1: DENSITY OF \hat{Z} IN MONTE CARLO EXPERIMENTS, NORMAL ERRORS, MEDIAN REGRESSION

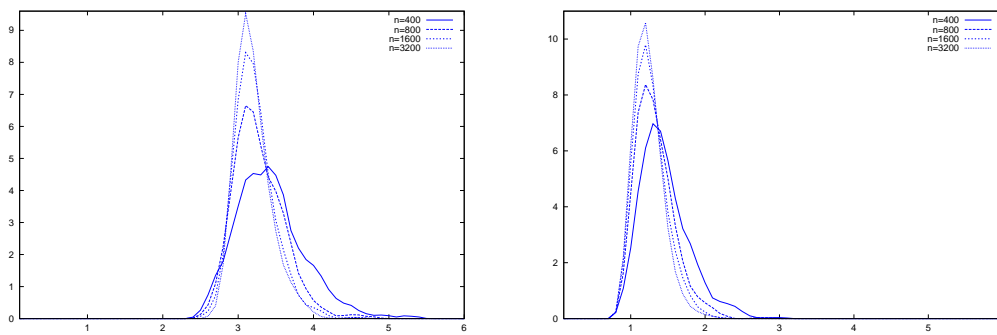


Figure 2: DENSITY OF \hat{Z} IN MONTE CARLO EXPERIMENTS, UNIFORM ERRORS, MEAN REGRESSION

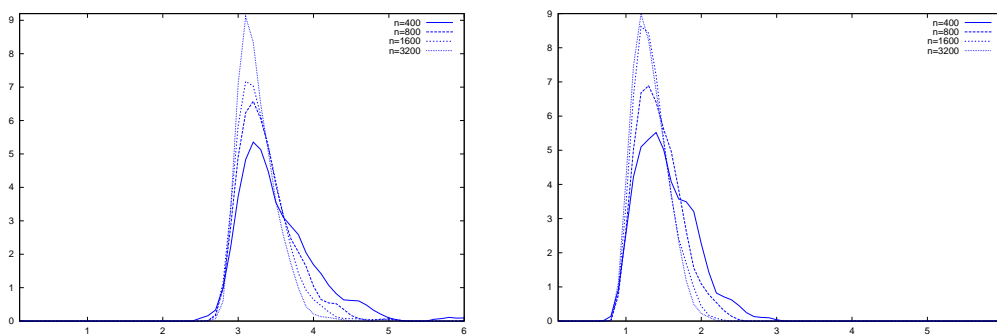


Figure 3: DENSITY OF \hat{Z} IN MONTE CARLO EXPERIMENTS, NORMAL ERRORS, MEAN REGRESSION

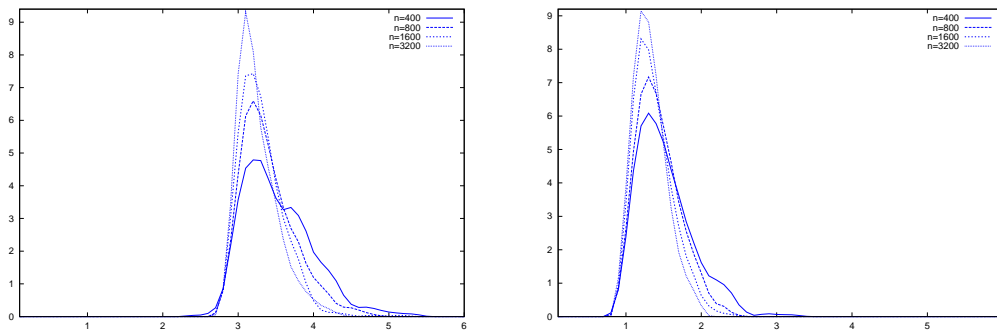


Figure 4: DENSITY OF \hat{Z} IN MONTE CARLO EXPERIMENTS, UNIFORM ERRORS, 9TH QUANTILE REGRESSION

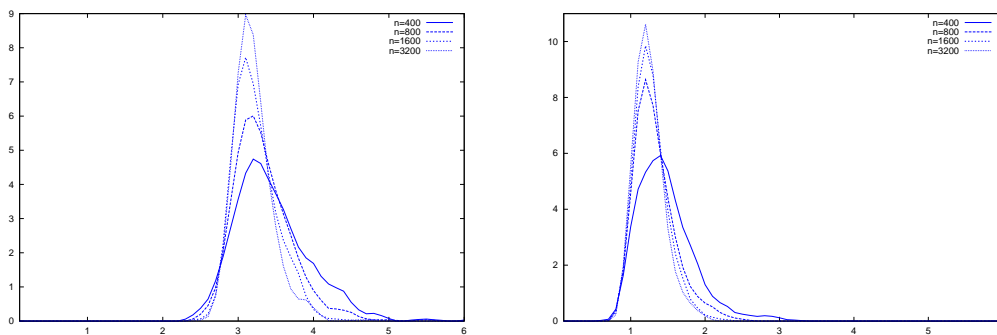


Table 1: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN MINORITY SHARE

place	70s		80s		90s		
	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .8$
Akron, OH PMSA	[0,1]	[0,0]	[1,1]	[1,1]	[1,2]	[0,1]	[0,1]
Albany-Schenectady-Troy, NY MSA	[0,1]	[0,1]	[2,3]	[1,3]	[0,1]	[0,1]	[0,1]
Albuquerque, NM MSA	[1,1]	[0,0]	[1,1]	[0,1]	[0,1]	[0,0]	[0,0]
Allentown-Bethlehem-Easton, PA MSA	[0,0]	[1,1]	[0,1]	[1,1]	[1,1]	[0,1]	[0,0]
Ann Arbor, MI PMSA	[1,2]	[2,3]	[0,1]	[1,1]	[1,1]	[0,0]	[0,0]
Atlanta, GA MSA	[1,1]	[1,1]	[2,3]	[0,0]	[0,0]	[0,0]	[0,0]
Austin-San Marcos, TX MSA	[1,1]	[0,0]	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]
Bakersfield, CA MSA	[0,0]	[0,1]	[0,1]	[0,0]	[1,1]	[1,1]	[1,1]
Baltimore, MD PMSA	[0,1]	[0,0]	[1,1]	[0,1]	[0,0]	[0,1]	[0,0]
Balton Rouge, LA MSA	[1,1]	[0,1]	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]
Bergen-Passaic, NJ PMSA	[0,1]	[0,1]	[0,0]	[0,1]	[0,1]	[0,1]	[0,0]
Birmingham, AL MSA	[0,1]	[1,1]	[1,1]	[0,0]	[0,1]	[0,0]	[0,0]
Boston, MA-NH PMSA	[0,1]	[0,1]	[0,1]	[0,1]	[1,1]	[0,0]	[0,1]
Bridgeport, CT PMSA	[1,1]	[1,1]	[1,1]	[0,1]	[0,1]	[0,1]	[0,1]
Buffalo-Niagara Falls, NY MSA	[0,0]	[0,1]	[1,1]	[0,1]	[1,2]	[0,0]	[0,1]
Charleston-North Charleston, SC MSA	[1,1]	[0,1]	[0,1]	[1,1]	[0,1]	[1,1]	[0,1]
Charlotte-Gastonia-Rock Hill, NC-SC MSA	[0,1]	[1,1]	[0,1]	[1,1]	[1,1]	[0,0]	[0,0]
Chicago, IL PMSA	[0,1]	[0,1]	[2,2]	[0,1]	[1,1]	[0,1]	[0,0]
Cincinnati, OH-KY-IN PMSA	[0,1]	[0,1]	[1,1]	[1,1]	[2,2]	[0,1]	[0,1]
Cleveland-Lorain-Elyria, OH PMSA	[0,0]	[0,1]	[1,1]	[0,1]	[2,2]	[0,1]	[0,1]

Table 2: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN MINORITY SHARE

place	70s		80s		90s	
	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .2$	$e = .5$
Colorado Springs, CO MSA	[1,1]	[0,0]	[0,0]	[0,1]	[0,0]	[1,1]
Columbia, SC MSA	[1,1]	[2,2]	[0,0]	[0,0]	[0,0]	[0,0]
Columbus, OH MSA	[1,1]	[0,0]	[0,0]	[0,1]	[0,1]	[0,1]
Dallas, TX PMSA	[1,2]	[1,1]	[0,0]	[0,0]	[0,0]	[0,1]
Dayton-Springfield, OH MSA	[1,1]	[0,0]	[0,0]	[0,0]	[0,1]	[0,1]
Denver, CO PMSA	[1,2]	[0,0]	[2,3]	[0,1]	[1,2]	[0,1]
Des Moines, IA MSA	[1,2]	[1,2]	[0,1]	[1,2]	[1,1]	[0,1]
Detroit, MI PMSA	[1,2]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
El Paso, TX MSA	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[1,1]
Flint, MI PMSA	[0,1]	[0,1]	[1,1]	[0,1]	[0,1]	[0,1]
Fort Lauderdale, FL PMSA	[0,1]	[1,1]	[1,2]	[0,0]	[0,0]	[0,0]
Fort Myers-Cape Coral, FL MSA	[0,0]	[0,0]	[0,1]	[3,5]	[0,1]	[0,0]
Fort Wayne, IN MSA	[1,2]	[0,0]	[1,2]	[1,1]	[0,1]	[0,0]
Fort Worth-Arlington, TX PMSA	[1,1]	[0,0]	[1,1]	[0,0]	[0,1]	[0,1]
Fresno, CA MSA	[0,0]	[0,0]	[1,1]	[0,1]	[0,0]	[1,1]
Gary, IN PMSA	[0,1]	[1,1]	[0,1]	[0,1]	[0,1]	[0,1]
Grand Rapids-Muskegon-Holland, MI MSA	[1,3]	[0,1]	[1,1]	[0,0]	[0,1]	[0,0]
Greenboro-Winston Salem-High Point, NC MSA	[0,1]	[1,1]	[1,1]	[0,1]	[0,0]	[0,1]
Greenville-Spartanburg-Anderson, SC MSA	[0,1]	[1,1]	[1,1]	[0,1]	[0,0]	[0,0]
Harrisburg-Lebanon-Carlisle, PA MSA	[0,1]	[0,0]	[0,0]	[0,1]	[0,0]	[0,1]

Table 3: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN MINORITY SHARE

place	70s		80s		90s		
	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .8$
Hartford, CT MSA	[0,1]	[0,1]	[1,1]	[0,1]	[0,1]	[0,1]	[0,1]
Honolulu, HI MSA	[0,1]	[0,0]	[1,1]	[1,1]	[0,1]	[0,0]	[0,0]
Houston, TX PMSA	[1,1]	[0,0]	[1,2]	[0,1]	[0,1]	[0,0]	[0,0]
Indianapolis, IN MSA	[1,2]	[0,0]	[0,1]	[1,1]	[2,2]	[0,1]	[0,0]
Jackson, MS MSA	[0,1]	[1,1]	[1,1]	[2,2]	[1,1]	[0,0]	[0,0]
Jacksonville, FL MSA	[1,1]	[1,2]	[1,1]	[0,1]	[1,1]	[0,1]	[0,0]
Jersey City, NJ PMSA	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]
Johnson City-Kingsport-Bristol, TN-VA MSA	[0,1]	[1,1]	[0,1]	[0,1]	[0,0]	[0,0]	[0,0]
Kalamazoo-Battle Creek, MI MSA	[1,2]	[0,0]	[1,1]	[0,0]	[1,1]	[0,0]	[0,0]
Kansas City, MO-KS MSA	[0,1]	[0,1]	[1,1]	[0,1]	[1,1]	[0,1]	[0,1]
Knoxville, TN MSA	[0,0]	[2,3]	[1,1]	[0,0]	[1,1]	[1,1]	[0,0]
Lakeland-Winter Haven, FL MSA	[1,2]	[1,1]	[1,1]	[2,2]	[1,2]	[0,0]	[0,0]
Lansing-East Lansing, MI MSA	[0,1]	[0,1]	[0,1]	[1,2]	[2,2]	[1,1]	[0,0]
Las Vegas, NV-AZ MSA	[1,1]	[0,0]	[1,1]	[1,1]	[1,1]	[1,1]	[0,0]
Lexington, KY MSA	[0,1]	[1,2]	[0,1]	[0,1]	[0,1]	[1,2]	[0,0]
Little Rock-North Little Rock, AR MSA	[1,1]	[0,0]	[0,1]	[0,1]	[1,1]	[0,1]	[0,0]
Los Angeles-Long Beach, CA PMSA	[1,1]	[1,1]	[0,1]	[0,1]	[1,1]	[1,1]	[0,0]
Louisville, KY-IN MSA	[1,1]	[0,0]	[1,1]	[0,1]	[0,1]	[0,1]	[0,1]
Memphis, TN-AR-MS MSA	[1,1]	[1,1]	[0,1]	[1,2]	[1,1]	[0,0]	[0,0]
Miami, FL PMSA	[0,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]

Table 4: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN MINORITY SHARE

place	70s			80s			90s		
	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$
Middlesex-Somerset-Hunterdon, NJ PMSA	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,1]	[0,1]	[0,0]
Milwaukee-Waukesha, WI PMSA	[0,1]	[0,1]	[1,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
Minneapolis-St. Paul, MN-WI MSA	[1,2]	[1,2]	[0,0]	[1,1]	[0,0]	[0,0]	[0,1]	[0,1]	[0,1]
Mobile, AL MSA	[1,1]	[1,2]	[0,0]	[1,1]	[0,0]	[0,0]	[1,1]	[0,1]	[0,0]
Momouth-Ocean, NJ PMSA	[1,1]	[1,1]	[0,0]	[0,1]	[1,1]	[0,0]	[1,1]	[0,0]	[0,0]
Nashville, TN MSA	[2,2]	[1,2]	[0,0]	[0,0]	[0,0]	[0,1]	[1,1]	[0,1]	[0,0]
Nassau-Suffolk, NY PMSA	[0,1]	[0,1]	[0,1]	[0,0]	[0,1]	[0,0]	[0,1]	[0,1]	[0,0]
New Haven-Meriden, CT PMSA	[0,1]	[0,1]	[0,1]	[2,3]	[0,0]	[0,0]	[1,1]	[0,1]	[0,0]
New Orleans, LA MSA	[1,1]	[0,0]	[0,0]	[1,1]	[1,1]	[0,1]	[1,1]	[0,1]	[0,0]
New York, NY PMSA	[0,1]	[0,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]
Newark, NJ PMSA	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,0]	[0,1]	[0,1]	[0,1]
Norfolk-Virginia Beach-Newport News, VA-NC MSA	[1,1]	[1,2]	[0,0]	[0,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]
Oakland, CA PMSA	[1,1]	[0,1]	[0,1]	[1,1]	[0,1]	[0,0]	[1,1]	[1,1]	[0,0]
Oklahoma City, OK MSA	[1,2]	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]	[1,1]	[1,1]	[0,0]
Omaha, NE-IA MSA	[0,0]	[0,1]	[0,0]	[1,1]	[0,1]	[0,1]	[1,1]	[0,1]	[0,0]
Orange County, CA PMSA	[0,0]	[0,1]	[0,0]	[0,1]	[0,1]	[0,0]	[0,1]	[0,1]	[0,0]
Orlando, FL MSA	[1,1]	[1,2]	[0,0]	[1,1]	[1,2]	[0,0]	[1,1]	[0,1]	[0,0]
Philadelphia, PA-NJ PMSA	[1,2]	[0,1]	[0,1]	[1,1]	[0,1]	[0,1]	[1,1]	[0,1]	[0,0]
Phoenix-Mesa, AZ MSA	[1,1]	[0,0]	[0,0]	[1,1]	[0,1]	[0,0]	[1,1]	[0,1]	[0,0]
Pittsburgh, PA MSA	[1,1]	[0,1]	[0,1]	[2,2]	[0,0]	[0,1]	[0,0]	[0,1]	[0,1]

Table 5: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN MINORITY SHARE

place	70s			80s			90s		
	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$
Portland-Vancouver, OR-WA PMSA	[0,0]	[0,1]	[0,0]	[0,0]	[2,3]	[1,2]	[0,1]	[1,2]	[0,1]
Providence-Fall River-Warwick, RI-MA MSA	[1,1]	[0,0]	[0,1]	[0,1]	[0,0]	[0,0]	[0,1]	[0,1]	[0,0]
Raleigh-Durham-Chapel Hill, NC MSA	[1,1]	[2,2]	[0,0]	[1,1]	[1,2]	[0,0]	[1,1]	[0,0]	[0,0]
Richmond-Petersburg, VA MSA	[0,1]	[1,1]	[0,0]	[0,1]	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]
Riverside-San Bernardino, CA PMSA	[0,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,1]	[0,0]	[0,0]
Rochester, NY MSA	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
Sacramento, CA PMSA	[1,2]	[1,1]	[0,0]	[0,0]	[1,1]	[0,0]	[1,2]	[0,1]	[0,0]
Saginaw-Bay City-Midland, MI MSA	[1,1]	[0,0]	[0,0]	[0,1]	[0,0]	[0,1]	[0,1]	[0,1]	[0,1]
St. Louis, MO-IL MSA	[0,0]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[1,1]	[0,1]	[0,1]
Salt Lake City-Ogden, UT MSA	[2,3]	[1,2]	[1,2]	[1,1]	[1,1]	[1,1]	[1,1]	[0,1]	[1,1]
San Antonio, TX MSA	[0,0]	[0,0]	[0,0]	[1,1]	[0,1]	[0,0]	[0,0]	[0,0]	[0,0]
San Diego, CA MSA	[0,1]	[0,1]	[0,0]	[0,1]	[0,1]	[0,0]	[0,0]	[0,0]	[0,0]
San Francisco, CA PMSA	[1,1]	[0,1]	[0,1]	[0,0]	[0,1]	[0,0]	[1,1]	[0,0]	[0,0]
San Jose, CA PMSA	[0,1]	[1,1]	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,1]	[1,1]
Sarasota-Bradenton, FL MSA	[0,1]	[1,1]	[0,1]	[1,1]	[1,2]	[0,0]	[1,2]	[0,0]	[0,0]
Scranton-Wilkes Barre-Hazleton, PA MSA	[0,1]	[0,1]	[0,1]	[0,1]	[0,0]	[0,0]	[0,1]	[0,1]	[0,1]
Seattle-Bellevue-Everett, WA PMSA	[0,1]	[1,2]	[0,0]	[0,1]	[1,2]	[0,0]	[1,1]	[0,0]	[0,0]
Spokane, WA MSA	[0,1]	[1,1]	[0,0]	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,1]
Springfield, MA MSA	[0,0]	[0,1]	[2,2]	[1,1]	[1,1]	[0,0]	[0,1]	[0,1]	[0,0]
Stockton-Lodi, CA MSA	[0,0]	[0,0]	[0,0]	[1,2]	[0,0]	[0,0]	[1,1]	[1,1]	[0,0]

Table 6: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN MINORITY SHARE

place	70s			80s			90s		
	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$
Syracuse, NY MSA	[0,0]	[0,1]	[0,1]	[2,2]	[1,1]	[0,1]	[0,1]	[0,1]	[0,1]
Tacoma, WA PMSA	[1,2]	[1,2]	[1,2]	[0,1]	[0,0]	[0,0]	[1,2]	[1,1]	[1,1]
Tampa-St. Petersburg-Clearwater, FL MSA	[1,1]	[0,1]	[0,0]	[0,0]	[0,1]	[0,0]	[1,1]	[0,1]	[0,0]
Toledo, OH MSA	[1,1]	[0,0]	[0,0]	[0,1]	[0,1]	[0,1]	[1,1]	[0,1]	[0,0]
Tucson, AZ MSA	[0,1]	[2,3]	[0,1]	[1,1]	[0,0]	[0,0]	[1,1]	[0,1]	[0,0]
Tulsa, OK MSA	[1,1]	[2,2]	[0,1]	[1,2]	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]
Vallejo-Fairfield-Napa, CA PMSA	[1,1]	[1,1]	[2,3]	[1,1]	[0,0]	[0,0]	[1,2]	[0,0]	[0,0]
Ventura, CA PMSA	[1,1]	[1,1]	[1,1]	[0,1]	[0,0]	[0,0]	[1,1]	[1,1]	[1,1]
Washington, DC-MD-VA-WV PMSA	[0,1]	[0,0]	[0,0]	[1,1]	[0,1]	[0,0]	[1,1]	[0,1]	[0,0]
West Palm Beach-Boca Raton, FL MSA	[0,1]	[1,1]	[1,3]	[1,1]	[0,0]	[0,0]	[1,1]	[0,1]	[0,0]
Wichita, KS MSA	[0,0]	[0,0]	[0,0]	[1,2]	[1,1]	[0,0]	[1,1]	[0,0]	[0,0]
Wilmington-Newark, DE-MD PMSA	[1,2]	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]
Worcester, MA-CT PMSA	[1,1]	[0,1]	[0,0]	[0,1]	[0,1]	[0,0]	[0,2]	[0,0]	[0,0]
Youngstown-Warren, OH MSA	[2,3]	[0,1]	[0,1]	[1,1]	[0,0]	[0,1]	[2,2]	[0,0]	[0,1]

Table 7: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN WHITE POPULATION

place	70s			80s			90s		
	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$
Akron, OH PMSA	[0,0]	[1,1]	[1,1]	[0,0]	[0,0]	[2,2]	[0,0]	[1,1]	[1,2]
Albany-Schenectady-Troy, NY MSA	[1,1]	[1,2]	[2,3]	[0,0]	[0,3]	[0,2]	[0,1]	[1,1]	[1,1]
Albuquerque, NM MSA	[0,1]	[1,2]	[1,1]	[0,0]	[0,1]	[0,0]	[0,1]	[0,0]	[0,0]
Allentown-Bethlehem-Easton, PA MSA	[0,0]	[1,1]	[0,1]	[0,0]	[0,1]	[1,2]	[0,0]	[1,1]	[1,2]
Ann Arbor, MI PMSA	[1,1]	[2,2]	[0,1]	[1,2]	[1,1]	[0,0]	[1,1]	[2,3]	[0,0]
Atlanta, GA MSA	[0,1]	[1,1]	[0,1]	[1,1]	[1,1]	[1,1]	[1,1]	[1,2]	[0,1]
Austin-San Marcos, TX MSA	[0,1]	[1,1]	[0,0]	[1,1]	[1,2]	[0,0]	[0,1]	[0,1]	[0,0]
Bakersfield, CA MSA	[0,1]	[1,2]	[1,1]	[1,2]	[2,2]	[0,0]	[0,1]	[1,1]	[0,1]
Baltimore, MD PMSA	[0,1]	[0,1]	[0,1]	[0,0]	[0,1]	[2,2]	[0,0]	[0,1]	[1,1]
Balton Rouge, LA MSA	[0,0]	[0,0]	[0,1]	[0,0]	[1,1]	[0,1]	[1,1]	[1,1]	[0,0]
Bergen-Passaic, NJ PMSA	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]	[0,0]	[0,1]	[1,1]	[1,1]
Birmingham, AL MSA	[0,0]	[1,2]	[0,0]	[0,0]	[1,1]	[1,2]	[0,1]	[1,2]	[1,1]
Boston, MA-NH PMSA	[0,1]	[0,1]	[0,1]	[0,0]	[0,0]	[1,1]	[0,0]	[0,1]	[0,1]
Bridgeport, CT PMSA	[0,1]	[0,1]	[1,1]	[0,0]	[0,0]	[1,1]	[0,0]	[1,1]	[3,4]
Buffalo-Niagara Falls, NY MSA	[0,0]	[0,0]	[0,2]	[0,0]	[0,0]	[1,3]	[0,0]	[0,0]	[3,4]
Charleston-North Charleston, SC MSA	[0,0]	[0,0]	[0,0]	[0,0]	[0,1]	[0,0]	[1,1]	[1,1]	[0,0]
Charlotte-Gastonia-Rock Hill, NC-SC MSA	[1,1]	[1,1]	[0,1]	[0,0]	[1,1]	[0,0]	[0,1]	[0,1]	[0,1]
Chicago, IL PMSA	[0,1]	[0,1]	[0,1]	[0,0]	[0,1]	[1,1]	[0,1]	[0,1]	[0,1]
Cincinnati, OH-KY-IN PMSA	[0,0]	[1,1]	[1,1]	[0,0]	[1,1]	[1,2]	[0,1]	[1,1]	[1,1]
Cleveland-Lorain-Elyria, OH PMSA	[0,0]	[0,1]	[1,1]	[0,0]	[0,0]	[0,2]	[0,1]	[0,1]	[1,1]

Table 8: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN WHITE POPULATION

place	70s		80s		90s	
	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .2$	$e = .5$
Colorado Springs, CO MSA	[1,1]	[1,1]	[2,2]	[1,1]	[0,0]	[0,1]
Columbia, SC MSA	[0,0]	[0,1]	[0,0]	[1,1]	[0,0]	[1,1]
Columbus, OH MSA	[0,1]	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]
Dallas, TX PMSA	[0,1]	[0,1]	[0,1]	[1,1]	[0,2]	[0,1]
Dayton-Springfield, OH MSA	[0,0]	[1,1]	[1,1]	[1,1]	[2,4]	[0,0]
Denver, CO PMSA	[0,1]	[0,1]	[0,1]	[1,1]	[0,1]	[0,1]
Des Moines, IA MSA	[0,1]	[1,2]	[3,3]	[1,3]	[2,3]	[1,1]
Detroit, MI PMSA	[0,1]	[0,1]	[0,1]	[0,0]	[1,1]	[0,1]
El Paso, TX MSA	[0,1]	[0,1]	[2,2]	[1,1]	[0,0]	[2,3]
Flint, MI PMSA	[0,0]	[1,1]	[0,0]	[1,1]	[0,1]	[1,1]
Fort Lauderdale, FL PMSA	[0,1]	[0,0]	[1,1]	[0,1]	[0,2]	[1,1]
Fort Myers-Cape Coral, FL MSA	[0,0]	[0,0]	[0,0]	[1,2]	[0,0]	[1,1]
Fort Wayne, IN MSA	[0,0]	[1,1]	[0,0]	[1,1]	[1,2]	[1,1]
Fort Worth-Arlington, TX PMSA	[0,1]	[1,2]	[0,0]	[1,2]	[0,0]	[1,2]
Fresno, CA MSA	[0,1]	[1,1]	[0,0]	[1,1]	[0,0]	[0,1]
Gary, IN PMSA	[0,0]	[1,1]	[2,3]	[1,1]	[1,1]	[1,1]
Grand Rapids-Muskegon-Holland, MI MSA	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[1,3]
Greenboro-Winston Salem-High Point, NC MSA	[0,0]	[0,1]	[0,1]	[1,1]	[0,1]	[1,2]
Greenville-Spartanburg-Anderson, SC MSA	[1,1]	[1,1]	[0,0]	[1,1]	[0,0]	[1,2]
Harrisburg-Lebanon-Carlisle, PA MSA	[0,0]	[2,2]	[1,2]	[1,1]	[0,2]	[1,1]

Table 9: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN WHITE POPULATION

place	70s		80s		90s		
	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .8$
Hartford, CT MSA	[0,1]	[0,1]	[0,0]	[1,1]	[0,0]	[1,1]	[1,1]
Honolulu, HI MSA	[0,1]	[1,1]	[0,0]	[0,1]	[0,0]	[0,0]	[1,1]
Houston, TX PMSA	[0,1]	[0,1]	[1,1]	[1,1]	[0,1]	[0,1]	[0,1]
Indianapolis, IN MSA	[0,0]	[1,1]	[0,0]	[1,1]	[0,1]	[1,2]	[0,0]
Jackson, MS MSA	[0,0]	[1,1]	[0,0]	[1,2]	[0,0]	[1,1]	[1,1]
Jacksonville, FL MSA	[0,1]	[1,1]	[0,0]	[1,1]	[0,1]	[1,2]	[1,1]
Jersey City, NJ PMSA	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[1,2]
Johnson City-Kingsport-Bristol, TN-VA MSA	[1,2]	[1,2]	[0,0]	[1,1]	[0,0]	[1,1]	[1,2]
Kalamazoo-Battle Creek, MI MSA	[0,0]	[1,1]	[0,0]	[1,1]	[0,1]	[1,2]	[0,1]
Kansas City, MO-KS MSA	[0,1]	[0,1]	[0,0]	[1,1]	[0,1]	[1,1]	[1,1]
Knoxville, TN MSA	[0,0]	[0,2]	[0,0]	[0,1]	[1,1]	[2,3]	[0,1]
Lakeland-Winter Haven, FL MSA	[1,2]	[0,0]	[1,1]	[1,2]	[0,0]	[1,1]	[0,0]
Lansing-East Lansing, MI MSA	[2,3]	[2,3]	[0,1]	[0,2]	[0,1]	[0,1]	[1,2]
Las Vegas, NV-AZ MSA	[1,1]	[1,2]	[1,1]	[1,1]	[1,1]	[1,1]	[0,1]
Lexington, KY MSA	[0,0]	[0,1]	[1,2]	[0,2]	[1,1]	[0,1]	[0,1]
Little Rock-North Little Rock, AR MSA	[0,0]	[1,1]	[0,1]	[1,1]	[1,1]	[1,2]	[0,1]
Los Angeles-Long Beach, CA PMSA	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
Louisville, KY-IN MSA	[0,0]	[0,1]	[0,0]	[0,0]	[0,1]	[1,1]	[1,1]
Memphis, TN-AR-MS MSA	[0,0]	[1,2]	[0,0]	[1,2]	[0,0]	[1,1]	[0,1]
Miami, FL PMSA	[0,1]	[0,1]	[0,0]	[0,0]	[1,1]	[1,1]	[1,1]

Table 10: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN WHITE POPULATION

place	70s			80s			90s		
	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$
Middlesex-Somerset-Hunterdon, NJ PMSA	[0,1]	[0,1]	[0,1]	[0,0]	[0,0]	[0,1]	[0,1]	[1,1]	[1,1]
Milwaukee-Waukesha, WI PMSA	[0,1]	[0,1]	[1,1]	[0,0]	[1,1]	[1,1]	[0,0]	[0,1]	[1,1]
Minneapolis-St., Paul, MN-WI MSA	[0,1]	[1,1]	[1,1]	[0,0]	[1,1]	[0,1]	[0,1]	[0,1]	[0,1]
Mobile, AL MSA	[0,0]	[1,1]	[0,0]	[0,0]	[0,1]	[0,1]	[0,0]	[1,1]	[0,0]
Monmouth-Ocean, NJ PMSA	[0,0]	[1,1]	[1,1]	[0,1]	[1,1]	[0,0]	[0,0]	[1,1]	[0,0]
Nashville, TN MSA	[0,0]	[1,1]	[0,0]	[1,1]	[1,1]	[0,0]	[1,1]	[1,1]	[0,1]
Nassau-Suffolk, NY PMSA	[0,0]	[0,0]	[1,2]	[0,1]	[0,1]	[0,0]	[0,0]	[0,1]	[1,1]
New Haven-Meriden, CT PMSA	[0,0]	[1,1]	[1,1]	[0,0]	[1,1]	[0,3]	[0,0]	[1,1]	[1,2]
New Orleans, LA MSA	[0,0]	[1,1]	[0,1]	[0,0]	[0,0]	[1,2]	[0,1]	[0,1]	[0,1]
New York, NY PMSA	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]
Newark, NJ PMSA	[0,0]	[0,0]	[1,1]	[0,0]	[0,0]	[1,1]	[0,1]	[1,1]	[1,2]
Norfolk-Virginia Beach-Newport News, VA-NC MSA	[0,1]	[1,2]	[0,0]	[0,0]	[2,2]	[0,0]	[0,1]	[1,1]	[0,1]
Oakland, CA PMSA	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[1,1]	[1,1]
Oklahoma City, OK MSA	[0,1]	[0,1]	[0,1]	[0,0]	[1,1]	[0,3]	[0,0]	[1,1]	[0,0]
Omaha, NE-IA MSA	[0,1]	[1,1]	[1,1]	[0,0]	[0,1]	[1,2]	[0,1]	[0,1]	[0,1]
Orange County, CA PMSA	[0,0]	[0,0]	[0,1]	[0,0]	[1,1]	[1,1]	[0,1]	[0,1]	[0,1]
Orlando, FL MSA	[0,0]	[0,1]	[0,1]	[0,0]	[1,1]	[0,0]	[0,1]	[1,1]	[0,1]
Philadelphia, PA-NJ PMSA	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[1,1]
Phoenix-Mesa, AZ MSA	[0,1]	[0,1]	[0,1]	[0,0]	[1,1]	[0,0]	[0,1]	[0,1]	[0,1]
Pittsburgh, PA MSA	[0,1]	[0,1]	[1,1]	[0,0]	[0,0]	[0,1]	[0,0]	[0,0]	[1,1]
Portland-Vancouver, OR-WA PMSA	[1,1]	[2,2]	[1,2]	[1,1]	[2,3]	[2,4]	[0,1]	[0,1]	[1,1]

Table 11: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN WHITE POPULATION

place	70s		80s		90s	
	$e = .2$	$e = .5$	$e = .2$	$e = .5$	$e = .2$	$e = .5$
Providence-Fall River-Warwick, RI-MA MSA	[0,0]	[1,1]	[0,0]	[1,1]	[0,0]	[1,1]
Raleigh-Durham-Chapel Hill, NC MSA	[0,1]	[0,1]	[1,1]	[1,2]	[0,1]	[0,1]
Richmond-Petersburg, VA MSA	[0,1]	[1,1]	[0,0]	[1,1]	[0,1]	[0,1]
Riverside-San Bernardino, CA PMSA	[1,1]	[1,2]	[0,0]	[0,1]	[0,1]	[0,1]
Rochester, NY MSA	[0,0]	[1,1]	[0,0]	[1,1]	[0,0]	[1,1]
Sacramento, CA PMSA	[1,1]	[1,1]	[1,1]	[0,1]	[0,1]	[0,1]
Saginaw-Bay City-Midland, MI MSA	[0,0]	[2,3]	[0,0]	[0,0]	[0,0]	[1,2]
St. Louis, MO-IL MSA	[0,1]	[1,1]	[0,0]	[1,1]	[0,1]	[1,1]
Salt Lake City-Ogden, UT MSA	[1,1]	[2,2]	[2,3]	[1,3]	[1,1]	[0,1]
San Antonio, TX MSA	[0,1]	[0,1]	[2,2]	[1,1]	[1,1]	[0,1]
San Diego, CA MSA	[0,1]	[0,1]	[0,1]	[1,1]	[0,1]	[0,1]
San Francisco, CA PMSA	[0,1]	[0,1]	[0,0]	[0,0]	[0,0]	[1,1]
San Jose, CA PMSA	[0,0]	[0,1]	[0,1]	[0,1]	[0,0]	[0,1]
Sarasota-Bradenton, FL MSA	[0,1]	[1,1]	[0,2]	[0,1]	[0,1]	[0,0]
Scranton-Wilkes Barre-Hazleton, PA MSA	[1,1]	[1,1]	[0,0]	[1,2]	[1,2]	[1,2]
Seattle-Bellevue-Everett, WA PMSA	[1,1]	[2,2]	[1,1]	[4,6]	[0,1]	[0,1]
Spokane, WA MSA	[0,0]	[1,1]	[0,0]	[1,1]	[1,1]	[1,1]
Springfield, MA MSA	[0,1]	[0,1]	[0,0]	[1,2]	[0,0]	[0,0]
Stockton-Lodi, CA MSA	[0,1]	[1,1]	[1,1]	[0,1]	[0,1]	[0,1]
Syracuse, NY MSA	[0,0]	[0,0]	[0,0]	[1,1]	[0,0]	[1,1]
Tacoma, WA PMSA	[2,2]	[2,2]	[0,1]	[0,1]	[1,2]	[2,2]

Table 12: .95 CONFIDENCE SETS FOR $Z(g)$ BY MSA, DECADE AND QUANTILE, CHANGE IN WHITE POPULATION

place	70s			80s			90s		
	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$	$e = .2$	$e = .5$	$e = .8$
Tampa-St. Petersburg-Clearwater, FL MSA	[0,1]	[0,1]	[0,1]	[0,1]	[1,2]	[0,0]	[0,1]	[0,1]	[0,1]
Toledo, OH MSA	[0,1]	[0,1]	[0,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,1]	[2,3]
Tucson, AZ MSA	[1,1]	[2,2]	[0,0]	[1,1]	[0,1]	[0,0]	[0,1]	[0,1]	[0,1]
Tulsa, OK MSA	[0,1]	[0,0]	[0,0]	[0,0]	[1,1]	[0,2]	[0,1]	[0,1]	[1,1]
Vallejo-Fairfield-Napa, CA PMSA	[1,1]	[1,1]	[0,0]	[0,1]	[1,1]	[0,0]	[0,0]	[1,1]	[1,2]
Ventura, CA PMSA	[1,1]	[1,2]	[1,1]	[0,0]	[1,1]	[0,0]	[0,0]	[0,1]	[0,1]
Washington, DC-MD-VA-WV PMSA	[0,1]	[0,0]	[0,1]	[0,0]	[1,1]	[0,0]	[0,1]	[0,1]	[0,1]
West Palm Beach-Boca Raton, FL MSA	[1,1]	[0,1]	[0,0]	[1,1]	[2,3]	[0,0]	[1,1]	[1,1]	[0,1]
Wichita, KS MSA	[0,0]	[1,1]	[1,2]	[0,0]	[1,1]	[1,1]	[0,1]	[0,1]	[0,1]
Wilmington-Newark, DE-MD PMSA	[0,0]	[0,1]	[0,1]	[0,0]	[2,3]	[0,0]	[0,0]	[1,1]	[0,1]
Worcester, MA-CT PMSA	[0,0]	[1,1]	[1,2]	[0,0]	[1,1]	[1,2]	[0,0]	[1,1]	[1,2]
Youngstown-Warren, OH MSA	[0,0]	[1,1]	[1,2]	[0,0]	[0,0]	[0,1]	[0,0]	[0,0]	[2,3]

4 Global dynamics of economic growth

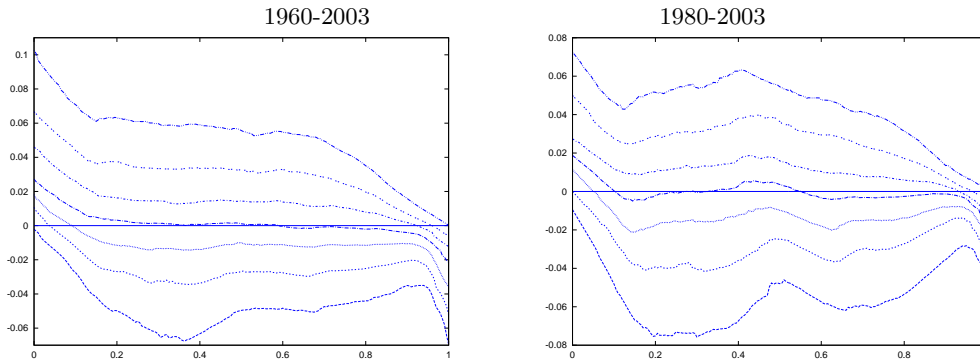
We will now apply the test for multiple roots to data on economic growth. The dataset used is the same commonly used in the empirical growth literature, see, e.g., Azariadis and Stachurski (2005), Quah (1996), or Durlauf, Johnson, and Temple (2005). The data are extracted from the Penn world tables, Heston, Summers, and Aten (2006) and Heston, Summers, and Aten (2009). The output measures used are real output per worker and real output per capita, RGDPWOK and RGDPCH. The dataset is restricted to the years after 1960, and to those countries for which output is available for every year in this time span. The variable Y is constructed as follows: For every year, output (per worker or per capita) is normalized by its average over the set of countries in the dataset, logarithmized and then transformed linearly to have range $[0, 1]$ over the pooled sample. The latter two steps are only for convenience and leave quantile regressions as well as their number of roots invariant, since they involve only monotonic transformations.

Figure 5 shows quantile regressions of ΔY on Y for the deciles 0.2 through 0.8, for output per capita and output per worker, and for the full time range as well as for only the years after 1980. Table 13 shows the corresponding confidence sets for the number of roots of these quantile regressions, as well as the number of unstable roots.

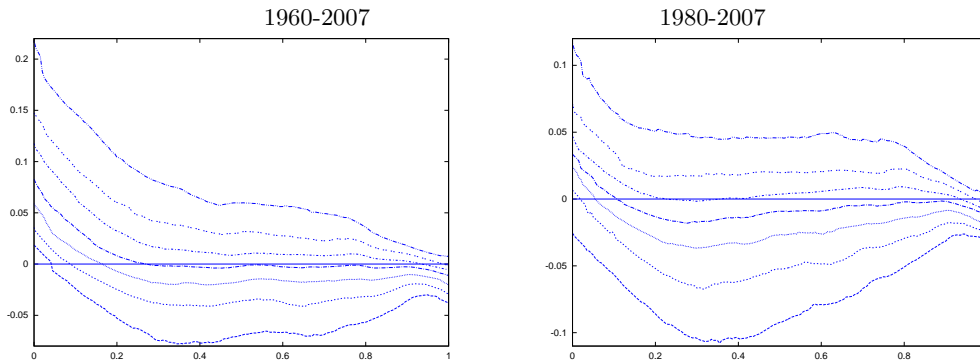
Both the graphs and the confidence sets for the full dataset, starting 1960, seem to suggest stable dynamics with one equilibrium. If the dataset is restricted to the years after 1980, in contrast, there does not seem to be convergence of incomes, and indeed there might be multiple roots at intermediate quantiles. Overall, however, evidence in favor of multiplicity of equilibria does not seem very strong. This stands in contrast to the bimodal ergodic distribution found by Quah (1996), which would seem to suggest “poverty trap” dynamics of economic growth.

These results have to be interpreted with great caution, however. First, the data might well be generated by underlying dynamics which are unstable in a higher dimensional state space even if dynamics in output alone appear to be stable. Second, exogenous factors might lead to different outcomes, so that the interesting dynamics are conditional on exogenous factors. Observability issues aside, these arguments would suggest to study the roots of quantile regressions of ΔY on Y as well as X , where Y is a higher dimensional endogenous vector and X summarizes exogenous factors. In the context of macro-level data this will soon lead to estimation problems, since we have many regressors but only a limited number of observations. Put differently, it is hard to find reasonable counterfactuals given a limited number of countries, an issue in macroeconometrics more generally. It might be interesting nevertheless to further explore the middle ground of controlling for only a few covariates and studying qualitative dynamics conditional on these covariates.

Figure 5: QUANTILE REGRESSIONS OF GROWTH ON INCOME, POST 1980
Output per worker



Output per capita



Notes: These graphs show local linear quantile regressions of ΔY on Y , where Y is log real GDP per capita relative to the global average, normalized linearly to lie in the interval $[0, 1]$, and ΔY is its annual change. The quantiles shown are the deciles 0.2 through 0.8.

Table 13: .95 CONFIDENCE SETS FOR $Z(g)$ AND $Z^u(g)$, WHERE g IS ESTIMATED BY QUANTILE REGRESSION OF GROWTH ON INCOME, OUTPUT PER CAPITA

output per worker

1960-2003		
quantile	Z	Z^u
0.2	[0,0]	[0,0]
0.3	[1,1]	[0,0]
0.4	[1,1]	[0,0]
0.5	[1,1]	[0,0]
0.6	[0,1]	[0,0]
0.7	[1,1]	[0,0]
0.8	[0,0]	[0,0]

1980-2003		
quantile	Z	Z^u
0.2	[0,0]	[0,0]
0.3	[0,1]	[0,0]
0.4	[1,1]	[0,0]
0.5	[3,3]	[1,1]
0.6	[1,1]	[0,0]
0.7	[1,1]	[0,0]
0.8	[0,1]	[0,0]

output per capita

1960-2007		
quantile	Z	Z^u
0.2	[0,0]	[0,0]
0.3	[1,1]	[0,0]
0.4	[0,1]	[0,0]
0.5	[2,2]	[0,0]
0.6	[0,1]	[0,0]
0.7	[0,1]	[0,0]
0.8	[0,0]	[0,0]

1980-2007		
quantile	Z	Z^u
0.2	[0,0]	[0,0]
0.3	[1,1]	[0,0]
0.4	[0,1]	[0,0]
0.5	[1,2]	[0,0]
0.6	[2,2]	[0,1]
0.7	[1,1]	[0,0]
0.8	[0,0]	[0,0]

Notes: The table shows confidence intervals in the integers for $Z(g)$ and $Z^u(g)$, where g is estimated by quantile regression of ΔY on Y . Y is log real GDP per worker (or per capita) relative to the global average and ΔY is its annual change. The bandwidth parameters are chosen as $\rho = 0.01$ and $\tau = 0.18$.

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