# Universal Adversarial Perturbations

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#### Introduction

• Universal adversarial perturbations are quasi-imperceptible perturbations designed to fool deep neural networks.

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- These perturbations are image-agnostic, meaning a single perturbation can cause misclassification across a wide range of images.

#### **Perturbed Images**



Figure 3: Examples of perturbed images and their corresponding labels. The first 8 images belong to the ILSVRC 2012 validation set, and the last 4 are images taken by a mobile phone camera. See supp. material for the original images.

#### Perturbations



Figure 4: Universal perturbations computed for different deep neural network architectures. Images generated with  $p = \propto \xi = 10$ . The pixel values are scaled for visibility.

#### **Perturbation Algorithm**

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#### • Parameters:

- $\xi$ : Magnitude of perturbation v.
- $\delta$ : Fooling rate threshold.

## **Perturbation Algorithm - Optimization**

• If current universal perturbation v does not fool data point  $x_i$ , seek the extra perturbation  $\Delta v_i$ 

$$\Delta v_i \leftarrow \arg\min_r \|r\|_2 \quad \text{s.t.} \ \hat{k}(x_i + v + r) \neq \hat{k}(x_i)$$

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Projection operator P<sub>p,ξ</sub>:

$$\mathcal{P}_{p,\xi}(v) = rg\min_{v'} \|v-v'\|_2 \quad ext{s.t.} \ \|v'\|_p \leq \xi$$

• Algorithm iterates until:

$$\mathsf{Err}(X_{\nu}) = \frac{1}{m} \sum_{i=1}^{m} \mathbb{1}_{\hat{k}(x_i+\nu) \neq \hat{k}(x_i)} \ge 1 - \delta$$

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• The number of datapoints *m* in *X* need not be large to compute a perturbation that is valid for the whole distribution!

#### Results

#### **Generalization across Data Points**

		CaffeNet [8]	VGG-F [2]	VGG-16 [17]	VGG-19 [17]	GoogLeNet [18]	ResNet-152 [6]
$\ell_2$	X	85.4%	85.9%	90.7%	86.9%	82.9%	89.7%
	Val.	85.6	87.0%	90.3%	84.5%	82.0%	88.5%
$\ell_{\infty}$	X	93.1%	93.8%	78.5%	77.8%	80.8%	85.4%
	Val.	93.3%	93.7%	78.3%	77.8%	78.9%	84.0%

Table 1: Fooling ratios on the set X, and the validation set.

• High fooling rates on set X as well as validation set (not used for computing v).

#### **Generalization across Architectures**

	VGG-F	CaffeNet	GoogLeNet	VGG-16	VGG-19	ResNet-152
VGG-F	93.7%	71.8%	48.4%	42.1%	42.1%	47.4 %
CaffeNet	74.0%	93.3%	47.7%	39.9%	39.9%	48.0%
GoogLeNet	46.2%	43.8%	78.9%	39.2%	39.8%	45.5%
VGG-16	63.4%	55.8%	56.5%	78.3%	73.1%	63.4%
VGG-19	64.0%	57.2%	53.6%	73.5%	77.8%	58.0%
ResNet-152	46.3%	46.3%	50.5%	47.0%	45.5%	84.0%

Table 2: Generalizability of the universal perturbations across different networks. The percentages indicate the fooling rates. The rows indicate the architecture for which the universal perturbations is computed, and the columns indicate the architecture for which the fooling rate is reported.

• Cross-model universality: Perturbations generalize well across different architectures!

## Explaining the Vulnerability to Universal Perturbations

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- Mean of the images (ImageNet bias).



Figure 8: Comparison between fooling rates of different perturbations. Experiments performed on the CaffeNet architecture.

• Suggests that decision boundaries of deep networks exhibit geometric correlations.

• For each image x in the validation set, compute the adversarial perturbation vector:

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- r(x) is normal to the decision boundary of the classifier at x + r(x).
- Define the matrix N, containing normalized vectors  $r(x_i)$ , as:

$$N = \begin{bmatrix} \frac{r(x_1)}{\|r(x_1)\|_2} & \frac{r(x_2)}{\|r(x_2)\|_2} & \cdots & \frac{r(x_n)}{\|r(x_n)\|_2} \end{bmatrix}$$

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• Compute the singular value decomposition (SVD) of N

$$N = U \Sigma V^T$$

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Figure 9: Singular values of matrix N containing normal vectors to the decision decision boundary.

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- Singular values of *N* decay quickly
- Suggests that a low-dimensional subspace captures most normal vectors to decision boundaries.

#### Low Dimensional Subspace



Figure 10: Illustration of the low dimensional subspace S containing normal vectors to the decision boundary in regions surrounding natural images. For the purpose of this illustration, we super-impose three data-points  $\{x_i\}_{i=1}^3$ , and the adversarial perturbations  $\{r_i\}_{i=1}^3$  that send the respective datapoints to the decision boundary  $\{\mathscr{B}_i\}_{i=1}^3$  are shown. Note that  $\{r_i\}_{i=1}^3$  all live in the subspace S.

#### **Conclusion & Discussion**

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Universal adversarial perturbations

- Exist for deep neural networks.
- Can generalize across images and different architectures.
- Decision boundaries show geometric correlations, allowing perturbations to exploit redundancies.

Future work

• A deeper theoretical analysis of the geometric properties of decision boundaries is needed to better understand these vulnerabilities.

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- Do humans have better 'training data'?
- Is the human brain just bigger in scale?
- Or is the way humans perceive images fundamentally different?

## Bluesky



- There are many interesting people on Bluesky!
- Explore starter packs across Machine Learning, AI, and Economics
  - Machine Learning Theory
  - ML & Probabilistic Stuff
  - Economists Working on AI
  - 'ML/AI People'
  - List of over 50 Econ Starter-Packs by Field